SCIENTIFIC OASIS

Spectrum of Mechanical Engineering and Operational Research

Journal homepage: www.smeor-journal.org
eISSN: 3042-0288

An Intelligent Decision Support System for Spherical Fuzzy Sugeno-Weber Aggregation Operators and Real-Life Applications

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ARTICLE INFO

Article history:
Received 2 April 2024
Received in revised form 11 July 2024
Accepted 31 July 2024
Available online 2 August 2024

Keywords:
Spherical Fuzzy Information; Sugeno-Weber Aggregation Operators; Decision Support System.

ABSTRACT

This article presents a novel approach to a decision support system for handling uncertainty and impreciseness in a large amount of human opinion. Sometimes, the aggregation of real-life applications is quite complex due to incomplete and redundant information about different preferences or alternatives. To handle such type of situations, a spherical fuzzy environment is a more effective and feasible framework with four components membership, abstinence, non-membership and refusal degree. We also formulate some flexible operations of Sugeno-Weber aggregation operators. Motivated by the theory of Sugeno-Weber t-norms, we constructed a family of mathematical methodologies, including Sugeno-Weber weighted average and weighted geometric operators in the light of spherical fuzzy information. An appropriate decision-making technique of the multi-attribute decision making (MADM) problem is also demonstrated to resolve complicated real-life applications. A numerical example is used to verify the compatibility and effectiveness of discussed mathematical approaches.

1. Introduction

The procedure of Multiple Attribute Decision Making (MADM) involves analyzing a small set of possibilities and ranking them according to how credible they are to the decision-maker(s) when all the regulations are considered. The rating estimates of each alternative in this process take into account both objective data and the subjective opinions of experts. Generally speaking, nevertheless, it is anticipated that the information they provided is current. In any case, genuine has several MADM difficulties where the data is unclear, missing, or of questionable quality due to the unpredictable nature of the framework step by step. The intuitionistic fuzzy set (IFS) framework was created by Atanassov [1] to handle it. It specified the degree of imprecision in terms of two functions, referred to as membership value (MV) and non-membership value (NMV), with the requirement that their sum does not exceed 1. The idea of the fuzzy set (FS) was developed by Zadeh [2], which defined the imprecision in an uncertain event with the aid of a single function known as the MV on a scale of 0

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https://doi.org/10.31181/smeor11202415

to 1, was improved by Atanassov's intuitionistic fuzzy model. Decision-makers are constrained by Atanassov's IFS in two ways. Firstly, it limits the total of MV and NMV up to a unit interval, making it impossible for decision-makers to award these grades on their own. Secondly, it is not applicable to situations where the viewpoint has a degree of abstinence in addition to refusal rather than being a yes-or-no type proposition. Nevertheless, the researchers employ theories like linguistic interval-valued IFS [3], linguistic interval-valued PyFS [4], q-rung orthopair fuzzy set (q-ROPFS) [5], cubic intuitionistic fuzzy set [6], Pythagorean fuzzy set (PyFS) [7], picture fuzzy set (PFS) [8], and interval-valued IFS [9] are employed in order to negotiate the ambiguities in the data. The advantages of these extended extensions are that they use MV, NMV, and the degree of hesitation to serve unclear information.

Since Atanassov's IFS has a very limited scope, it cannot be used in circumstances when there are more than two possible outcomes, such as yes, abstain, no, and refuse. For this reason, Cuong [10] presented the picture fuzzy set (PFS), a generalized model in FS theory that describes four features of an imprecise event. For PFS MV can be used to reflect the degree of favoritism, abstinence, dislike, and rejection that human opinion can have while making decisions. Similar to Atanassov's IFS, Cuong's PFS limits the decision maker from establishing the MVs by their own consent in some way by stating that the sum of the three MVs cannot exceed 1. Mahmood *et al*. [11] established a novel theory of the spherical fuzzy set (SFS) by expanding the range of PFS and suggested a solution for the limiting structure of PFS. The notion of SFS is expanded upon to encompass the concept of a t-spherical fuzzy set (T-SFS) with the addition of a parameter t, which allows decision-makers to select MVs from any point within the interval $[0,1]$. The most generic fuzzy framework available is the T-SFS to describe human opinion on any imprecise occurrence in a flexible and limitless manner.

In fuzzy mathematics, MADM is a prominent topic of study. Many fuzzy frameworks handle this subject, which is considered among the most important. The MADM process is typically facilitated by aggregation operators and, in certain situations, distance and similarity metrics. Numerous aggregation operators have already been produced in various environments. For instance, Xu and Yager [12-13] provided modified mathematical approaches of weighted averaging (WA) and weighted geometric (WG) operators to aggregate information based on IFS and PyFS environments. Using the ranking approach, Peng and Yang [14] investigated new PyFS operations like division and subtraction to determine superiority and inferiority in MADM situations. Hussain *et al.* [15] constructed hamy mean models taking into account different properties of Dombi t-norms. Ren *et al*. [16] established the concept of the pythagorean fuzzy TODIM method for selecting reliable optimal options under various criteria or attributes. By merging Einstein operations, Garg [17] examined various mathematical approaches by generalizing the qualities of pythagorean fuzzy frameworks. In order to address the MADM difficulties, Kaur and Garg [18] proposed a few operators for cubic IFS based on the t-norm. Yang *et al*. [19] constructed an online purchasing MADM algorithm by utilizing Heronian mean operators. However, Liu and Wang [20] defined the WA and WG operators within the q-ROPFS framework and examined their applications in MADM. Yang *et al*. [21] used the MADM approach using q-rung orthopair fuzzy information to further study the various heterogeneous associations.

Hussain *et al.* [22] proposed aggregation operators of Aczel Alsina t-norms to select efficient results by the available options. We also examined various mathematical approaches and fuzzy terminologies seen in [23-26]. The WA and geometric operators were defined by Garg [27] in the context of the PFS environment, while Wang *et al*. [28] offered several geometric operators to address decision-making issues. In contrast, Ullah *et al*. [29] introduced these operators to the T-SFS. In MADM problems, the applicability of various aggregation operations is also discussed. Hussain *et* *al.* [30] evaluated various type of solar panels based on multiple features and manufacturing materials.

The above-mentioned operators and measures have all been heavily employed in resolving the MADM issues. These studies do, however, reveal that all these algorithms are predicated on the idea that the traits under consideration are unrelated to one another. However, it is abundantly clear that in our everyday lives, every trait is immediately impacted by others. Consider a decision-making problem involving the purchase of a house, for example. The corresponding factors or attributes, such as features, price, and location, directly depend on one another. As a result, these properties are not taken into account in the method mentioned above. Stated differently, none of the aforementioned operators take into account the relationship between the values being used. Hussain *et al.* [31] aggregated some reliable suppliers using prioritization among different criteria or attribute information. Yager [32] defined the notion of a power aggregation operator in order to get around this problem. The relationship between the information being aggregated is important to power aggregation operators. A number of authors have used power aggregation operators to solve different MADM issues because of their significance. In a fuzzy context, Xu and Yager [33] presented some power WG aggregation operators. Fuzzy power aggregation operators were used by Wei *et al*. [34] to resolve the MADM issue. The power WA and WG operators for PyFS characteristics are expressed by Wei and Lu [35]. These power operators with IFS properties were studied by Xu [36].

Theory of Sugeno-Weber t-norms developed by two scientists Sugeno [37] and Weber [38]. Later on, different research scholars utilized proposed theories of Sugeno-Weber t-norms to resolve real-life applications [39-41]. The spherical fuzzy set is a more applicable discipline in the fuzzy field that is used to capture more tangible pictures by uncertain human opinions and redundant information. Motivated by the importance of the decision analysis process and spherical fuzzy framework, we constructed feasible mathematical strategies and decision algorithms for the MADM problem as follows:

a) To demonstrate basic notions of the spherical fuzzy framework with some necessary operations and properties.
b) Some feasible operations of Sugeno-Weber t-norms are formulated considering the spherical fuzzy environment.
c) We derived a family of robust mathematical approaches, namely spherical fuzzy Sugeno-Weber WA (SFSWWA) and spherical fuzzy Sugeno-Weber weighted geometric (SFSWWG) operators, to fuse complicated and insufficient information related to human opinion.
d) A few basic properties also validate the effectiveness and robustness of derived theories and aggregation operators.
e) The numerical examples illustrate the supremacy and applicability of the mathematical theories and decision algorithms of the MADM problem.

The remaining sections of this article are organized as follows: section 2 discussed basic notions of Sugeno-Weber t-norms and spherical fuzzy sets with necessary operations. Section 3 demonstrates research methodologies of Sugeno-Weber aggregation operators in the light of a spherical fuzzy environment. Section 4 establishes a robust decision-making technique for the MADM problem to validate deduced mathematical methodologies and real-life applications. A case study is also discussed to select a suitable optimal option under different conflicting criteria and attribute information using derived aggregation operators. Section 5 describes the whole presentation in a few comments.

2. Basic Concepts

This section demonstrates basic concepts related to the presented research work.

Definition 1: [42] The shape of the Sugeno-Weber t-norm and t-conorms are expressed as follows:

$$\mathring{R}^{T} = \begin{cases} \mathring{R}_D\left(\bar{\bar{a}},\bar{\bar{\vartheta}}\right) & if\ T = -1 \\ max\left(0,\dfrac{\bar{\bar{a}} + \bar{\bar{\vartheta}} - 1 + T\bar{\bar{a}}\bar{\bar{\vartheta}}}{1+T}\right) & if\ -1 < T < +\infty \\ \mathring{R}_P\left(\bar{\bar{a}},\bar{\bar{\vartheta}}\right) & T = +\infty \end{cases}$$

and

$$\dot{S}^{T} = \begin{cases} \dot{S}_D\left(\bar{\bar{a}},\bar{\bar{\vartheta}}\right) & if\ T = -1 \\ min\left(1,\bar{\bar{a}} + \bar{\bar{\vartheta}} - \dfrac{T}{1+T}\bar{\bar{a}}\,\bar{\bar{\vartheta}}\right) & if\ -1 < T < +\infty \\ \dot{S}_P\left(\bar{\bar{a}},\bar{\bar{\vartheta}}\right) & T = +\infty \end{cases}$$

Definition 2: [43] A SFS $\bar{\bar{A}}$ on a non-empty set $\hat{W}$ is given by:

$$\bar{\bar{A}} = \left\{a,\left(\bar{\bar{\mathcal{U}}}(a),\bar{\bar{\mathcal{V}}}(a),\bar{\bar{\mathcal{Y}}}(a)\right)|a \in \hat{W}\right\}$$

Noted that $\bar{\bar{\mathcal{U}}}: \hat{W} \to [0,1]$, $\bar{\bar{\mathcal{V}}}: \hat{W} \to [0,1]$ and $\bar{\bar{\mathcal{Y}}}: \hat{W} \to [0,1]$ denote the membership value, neutral value, and non-membership value. The SFS must fulfil the following conditions:

$$0 \le \overline{\overline{\mathcal{U}^2}}(a) + \overline{\overline{\mathcal{V}^2}}(a) + \overline{\overline{\mathcal{Y}^2}}(a) \le 1$$

The refusal index of a SFS is given by $r(a) = \sqrt{1-\left(\overline{\overline{\mathcal{U}^2}}(a) + \overline{\overline{\mathcal{V}^2}}(a) + \overline{\overline{\mathcal{Y}^2}}(a)\right)}$. A spherical fuzzy value (SFV) is denoted by $\bar{\bar{\mathfrak{D}}} = (\bar{\bar{\mathcal{U}}},\bar{\bar{\mathcal{V}}},\bar{\bar{\mathcal{Y}}})$.

Definition 3: [43] For three SFVs $\bar{\bar{\mathfrak{D}}} = \left(\bar{\bar{\mathcal{U}}}(a),\bar{\bar{\mathcal{V}}}(a),\bar{\bar{\mathcal{Y}}}(a)\right)$, $\overline{\overline{\mathfrak{D}_1}} = \left(\bar{\bar{\mathcal{U}}}_1(a),\overline{\overline{\mathcal{V}_1}}(a),\bar{\bar{\mathcal{Y}}}_1(a)\right)$ and $\overline{\overline{\mathfrak{D}_2}} = \left(\overline{\overline{\mathcal{U}_2}}(a),\overline{\overline{\mathcal{V}_2}}(a),\overline{\overline{\mathcal{Y}_2}}(a)\right)$. Then:

1) $\overline{\overline{\mathfrak{D}_1}} \subseteq \overline{\overline{\mathfrak{D}_2}}$, if $\bar{\bar{\mathcal{U}}}_1 \le \bar{\bar{\mathcal{U}}}_2, \overline{\overline{\mathcal{V}_1}} \ge \overline{\overline{\mathcal{V}_2}}$ and $\overline{\overline{\mathcal{Y}_1}} \ge \overline{\overline{\mathcal{Y}_2}}$
2) $\overline{\overline{\mathfrak{D}_1}} = \overline{\overline{\mathfrak{D}_2}}$, if $\overline{\overline{\mathfrak{D}_1}} \subseteq \overline{\overline{\mathfrak{D}_2}}$ and $\overline{\overline{\mathfrak{D}_1}} \supseteq \overline{\overline{\mathfrak{D}_2}}$
3) $\overline{\overline{\mathfrak{D}_1}} \cup \overline{\overline{\mathfrak{D}_2}} = \left\{max\left(\bar{\bar{\mathcal{U}}}_1(a),\bar{\bar{\mathcal{U}}}_2(a)\right), min\left(\overline{\overline{\mathcal{V}_1}}(a),\overline{\overline{\mathcal{V}_2}}(a)\right), min\left(\overline{\overline{\mathcal{Y}_1}}(a),\overline{\overline{\mathcal{Y}_2}}(a)\right) |(a) \in \hat{W}\right\}$
4) $\overline{\overline{\mathfrak{D}_1}} \cap \overline{\overline{\mathfrak{D}_2}} = \left\{min\left(\bar{\bar{\mathcal{U}}}_1(a),\bar{\bar{\mathcal{U}}}_2(a)\right), max\left(\overline{\overline{\mathcal{V}_1}}(a),\overline{\overline{\mathcal{V}_2}}(a)\right), max\left(\overline{\overline{\mathcal{Y}_1}}(a),\overline{\overline{\mathcal{Y}_2}}(a)\right) |(a) \in \hat{W}\right\}$
5) $\bar{\bar{\mathfrak{D}}}^C = \left(\bar{\bar{\mathcal{Y}}}(a),\bar{\bar{\mathcal{V}}}(a),\bar{\bar{\mathcal{U}}}(a)\right), \forall\ (a) \in \hat{W}$

Definition 4: [43] For three SFVs $\bar{\bar{\mathfrak{D}}} = (\bar{\bar{\mathcal{U}}},\bar{\bar{\mathcal{V}}},\bar{\bar{\mathcal{Y}}})$, $\overline{\overline{\mathfrak{D}_1}} = (\bar{\bar{\mathcal{U}}}_1,\overline{\overline{\mathcal{V}_1}},\overline{\overline{\mathcal{Y}_1}})$ and $\overline{\overline{\mathfrak{D}_2}} = (\overline{\overline{\mathcal{U}_2}},\overline{\overline{\mathcal{V}_2}},\overline{\overline{\mathcal{Y}_2}})$ with $₵ > 0$. Then, we have:

1) $\overline{\overline{\mathfrak{D}_1}} \oplus \overline{\overline{\mathfrak{D}_2}} = \left(\sqrt{\overline{\overline{\mathcal{U}^2}}_1 + \overline{\overline{\mathcal{U}^2}}_2 - \bar{\bar{\mathcal{U}}}^2{}_1.\overline{\overline{\mathcal{U}^2}}_2}, \overline{\overline{\mathcal{V}_1}}.\overline{\overline{\mathcal{V}_2}}, \overline{\overline{\mathcal{Y}_1}}.\overline{\overline{\mathcal{Y}_2}}\right)$
2) $\overline{\overline{\mathfrak{D}_1}} \otimes \overline{\overline{\mathfrak{D}_2}} = \left(\bar{\bar{\mathcal{U}}}_1.\bar{\bar{\mathcal{U}}}_2, \sqrt{\overline{\overline{\mathcal{V}_1^2}} + \overline{\overline{\mathcal{V}_2^2}} - \overline{\overline{\mathcal{V}_1^2}}.\overline{\overline{\mathcal{V}_2^2}}}, \sqrt{\overline{\overline{\mathcal{Y}_1^2}} + \overline{\overline{\mathcal{Y}_2^2}} - \overline{\overline{\mathcal{Y}_1^2}}.\overline{\overline{\mathcal{Y}_2^2}}}\right)$

3) $\text{¢}\overline{\overline{\mathfrak{D}}} = \left(\sqrt{1-\left(1-\overline{\overline{\mathcal{U}^2}}\right)^{\text{¢}}}, \bar{\bar{\mathcal{V}}}^{\text{¢}}, \bar{\bar{\mathcal{Y}}}^{\text{¢}}\right)$

4) $\overline{\overline{\mathfrak{D}}}^{\text{¢}} = \left(\bar{\bar{\mathcal{U}}}^{\text{¢}}, \sqrt{1-\left(1-\overline{\overline{\mathcal{V}^2}}\right)^{\text{¢}}}, \sqrt{1-\left(1-\overline{\overline{\mathcal{Y}^2}}\right)^{\text{¢}}}\right)$

Definition 5: [43] Let a SFV $\overline{\overline{\mathfrak{D}}} = \left(\bar{\bar{\mathcal{U}}}, \bar{\bar{\mathcal{V}}}, \bar{\bar{\mathcal{Y}}}\right)$ and score function of $\overline{\overline{\mathfrak{D}}}$ is expressed as follows:

$$\mathcal{S}\left(\overline{\overline{\mathfrak{D}}}\right) = \frac{1}{3}\left(\overline{\overline{\mathcal{U}^2}}(a) - \overline{\overline{\mathcal{V}^2}}(a) - \overline{\overline{\mathcal{Y}^2}}(a)\right), \mathcal{S}\left(\overline{\overline{\mathfrak{D}}}\right) \in [-1,1]$$

3. Research Methodologies for Spherical Fuzzy Information

This section constructed a family of robust mathematical strategies using Sugeno-Weber operations and spherical fuzzy information.

Definition 6: For three SFVs $\overline{\overline{\mathfrak{D}}} = \left(\bar{\bar{\mathcal{U}}}, \bar{\bar{\mathcal{V}}}, \bar{\bar{\mathcal{Y}}}\right), \overline{\overline{\mathfrak{D}_1}} = \left(\bar{\bar{\mathcal{U}}}_1, \overline{\overline{\mathcal{V}_1}}, \overline{\overline{\mathcal{Y}_1}}\right)$ and $\overline{\overline{\mathfrak{D}_2}} = \left(\overline{\overline{\mathcal{U}_2}}, \overline{\overline{\mathcal{V}_2}}, \overline{\overline{\mathcal{Y}_2}}\right)$ with $\Delta > 0$. So, we have:

a) $$\overline{\overline{\mathfrak{D}_1}} \oplus \overline{\overline{\mathfrak{D}_2}} = \begin{pmatrix} \sqrt{\overline{\overline{\mathcal{U}_1^2}} + \overline{\overline{\mathcal{U}_2^2}} - \frac{\mathcal{T}}{1+\mathcal{T}}\overline{\overline{\mathcal{U}_1^2}}.\overline{\overline{\mathcal{U}_2^2}}}, \\ \sqrt{\frac{\overline{\overline{\mathcal{V}_1^2}} + \overline{\overline{\mathcal{V}_2^2}} - 1 + \mathcal{T}\overline{\overline{\mathcal{V}_1^2}}.\overline{\overline{\mathcal{V}_2^2}}}{1+\mathcal{T}}}, \\ \sqrt{\frac{\overline{\overline{\mathcal{Y}_1^2}} + \overline{\overline{\mathcal{Y}_2^2}} - 1 + \mathcal{T}\overline{\overline{\mathcal{Y}_1^2}}.\overline{\overline{\mathcal{Y}_2^2}}}{1+\mathcal{T}}} \end{pmatrix}$$

b) $$\overline{\overline{\mathfrak{D}_1}} \otimes \overline{\overline{\mathfrak{D}_2}} = \begin{pmatrix} \sqrt{\frac{\overline{\overline{\mathcal{U}_1^2}} + \overline{\overline{\mathcal{U}_2^2}} - 1 + \mathcal{T}\overline{\overline{\mathcal{U}_1^2}}.\overline{\overline{\mathcal{U}_2^2}}}{1+\mathcal{T}}}, \\ \sqrt{\overline{\overline{\mathcal{V}_1^2}} + \overline{\overline{\mathcal{V}_2^2}} - \frac{\mathcal{T}}{1+\mathcal{T}}\overline{\overline{\mathcal{V}_1^2}}.\overline{\overline{\mathcal{V}_2^2}}}, \\ \sqrt{\overline{\overline{\mathcal{Y}_1^2}} + \overline{\overline{\mathcal{Y}_2^2}} - \frac{\mathcal{T}}{1+\mathcal{T}}\overline{\overline{\mathcal{Y}_1^2}}.\overline{\overline{\mathcal{Y}_2^2}}} \end{pmatrix}$$

c) $$\Delta\overline{\overline{\mathfrak{D}}} = \begin{pmatrix} \sqrt{\frac{1+\mathcal{T}}{\mathcal{T}}\left(1-\left(1-\overline{\overline{\mathcal{U}^2}}\left(\frac{\mathcal{T}}{1+\mathcal{T}}\right)\right)^{\Delta}\right)}, \\ \sqrt{\left((1+\mathcal{T})\left(\frac{\mathcal{T}\overline{\overline{\mathcal{V}^2}}+1}{1+\mathcal{T}}\right)^{\Delta} - 1\right)\frac{1}{\mathcal{T}}}, \\ \sqrt{\left((1+\mathcal{T})\left(\frac{\mathcal{T}\overline{\overline{\mathcal{Y}^2}}+1}{1+\mathcal{T}}\right)^{\Delta} - 1\right)\frac{1}{\mathcal{T}}} \end{pmatrix}$$

d) $$\overline{\overline{\mathfrak{D}^{\Delta}}} = \begin{pmatrix} \sqrt{\frac{1}{\mathcal{T}}\left((1+\mathcal{T})\left(\frac{\mathcal{T}\overline{\overline{\mathcal{U}^2}}+1}{1+\mathcal{T}}\right)^{\Delta} - 1\right)}, \\ \sqrt{\frac{1+\mathcal{T}}{\mathcal{T}}\left(1-\left(1-\overline{\overline{\mathcal{V}^2}}\left(\frac{\mathcal{T}}{1+\mathcal{T}}\right)\right)^{\Delta}\right)}, \\ \sqrt{\frac{1+\mathcal{T}}{\mathcal{T}}\left(1-\left(1-\overline{\overline{\mathcal{Y}^2}}\left(\frac{\mathcal{T}}{1+\mathcal{T}}\right)\right)^{\Delta}\right)} \end{pmatrix}$$

Definition 7: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right), \tau = 1, 2, \dots, \tilde{n}$, and the SFSWWA operator is given by:

$$SFSWWA\left(\overline{\overline{\mathfrak{D}_1}}, \overline{\overline{\mathfrak{D}_2}}, \dots, \overline{\overline{\mathfrak{D}_{ñ}}}\right) = \bigoplus_{\tau=1}^{ñ} ш_\tau \overline{\overline{\mathfrak{D}_\tau}}$$

Where $ш = (ш_1, ш_2, \dots, ш_{ñ})$ be the set of weight vector of $\overline{\overline{\mathfrak{D}_\tau}}$ such that $ш_\tau > 0$ and $\sum_{\tau=1}^{ñ} ш_\tau = 1$.

Theorem 1: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right), \tau = 1, 2, \dots, ñ$, and the integrated value of the SFSWWA operator is still a SFV, so we have:

$$SFSWWA\left(\overline{\overline{\mathfrak{D}_1}}, \overline{\overline{\mathfrak{D}_2}}, \dots, \overline{\overline{\mathfrak{D}_{ñ}}}\right) = \begin{pmatrix} \sqrt{\frac{1+\mathcal{T}}{\mathcal{T}}\left(1 - \prod_{\tau=1}^{ñ}\left(1 - \overline{\overline{\mathcal{U}_\tau^2}}\left(\frac{\mathcal{T}}{1+\mathcal{T}}\right)\right)^{ш_\tau}\right)}, \\ \sqrt{\frac{1}{\mathcal{T}}\left((1+\mathcal{T})\prod_{\tau=1}^{ñ}\left(\frac{\mathcal{T}\overline{\overline{\mathcal{V}_\tau^2}}+1}{1+\mathcal{T}}\right)^{ш_\tau} - 1\right)}, \\ \sqrt{\frac{1}{\mathcal{T}}\left((1+\mathcal{T})\prod_{\tau=1}^{ñ}\left(\frac{\mathcal{T}\overline{\overline{\mathcal{Y}_\tau^2}}+1}{1+\mathcal{T}}\right)^{ш_\tau} - 1\right)} \end{pmatrix}$$

Proof is analogous.

Theorem 2: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right), \tau = 1, 2, \dots, ñ$, implies that $\overline{\overline{\mathfrak{D}_\tau}} = \bar{\bar{\mathfrak{D}}}$. Then we have:

$$SFSWWA\left(\overline{\overline{\mathfrak{D}_1}}, \overline{\overline{\mathfrak{D}_2}}, \dots, \overline{\overline{\mathfrak{D}_{ñ}}}\right) = \bar{\bar{\mathfrak{D}}}$$

Theorem 3: For any two sets of a SFVs $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right)$ and $\overline{\overline{\mathfrak{D}'_\tau}} = \left(\bar{\bar{\mathcal{U}}}'_\tau, \overline{\overline{\mathcal{V}'_\tau}}, \overline{\overline{\mathcal{Y}'_\tau}}\right), \tau = 1, 2, \dots, ñ$. If $\overline{\overline{\mathfrak{D}_\tau}} \le \overline{\overline{\mathfrak{D}'_\tau}}$, so we have:

$$SFSWWA\left(\overline{\overline{\mathfrak{D}_1}}, \overline{\overline{\mathfrak{D}_2}}, \dots, \overline{\overline{\mathfrak{D}_{ñ}}}\right) \le SFSWWA\left(\overline{\overline{\mathfrak{D}'_1}}, \overline{\overline{\mathfrak{D}'_2}}, \dots, \overline{\overline{\mathfrak{D}'_{ñ}}}\right)$$

Theorem 4: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right), \tau = 1, 2, \dots, ñ$. If $\overline{\overline{\mathfrak{D}^-}} = \left(\min\{\overline{\overline{\mathcal{U}_\tau}}\}, \max\{\overline{\overline{\mathcal{V}_\tau}}\}, \max\{\overline{\overline{\mathcal{Y}_\tau}}\}\right)$ and $\overline{\overline{\mathfrak{D}^+}} = \left(\max\{\overline{\overline{\mathcal{U}_\tau}}\}, \min\{\overline{\overline{\mathcal{V}_\tau}}\}, \min\{\overline{\overline{\mathcal{Y}_\tau}}\}\right)$. Then we have:

$$\overline{\overline{\mathfrak{D}^-}} \le SFSWWA\left(\overline{\overline{\mathfrak{D}_1}}, \overline{\overline{\mathfrak{D}_2}}, \dots, \overline{\overline{\mathfrak{D}_{ñ}}}\right) \le \overline{\overline{\mathfrak{D}^+}}$$

Definition 8: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right), \tau = 1, 2, \dots, ñ$, and the SFSWPWA operator is characterized as follows:

$$SFSWWG\left(\overline{\overline{\mathfrak{D}_1}}, \overline{\overline{\mathfrak{D}_2}}, \dots, \overline{\overline{\mathfrak{D}_{ñ}}}\right) = \bigotimes_{\tau=1}^{ñ} \overline{\overline{\mathfrak{D}_\tau}}^{ш_\tau}$$

Where $ш = (ш_1, ш_2, \dots, ш_{ñ})$ be the set of weight vector of $\overline{\overline{\mathfrak{D}_\tau}}$ such that $ш_\tau > 0$ and $\sum_{\tau=1}^{ñ} ш_\tau = 1$.

Theorem 5: As $\overline{\overline{\mathfrak{D}_\tau}} = \left(\bar{\bar{\mathcal{U}}}_\tau, \overline{\overline{\mathcal{V}_\tau}}, \overline{\overline{\mathcal{Y}_\tau}}\right), \tau = 1, 2, \dots, ñ$ be a class of SFVs, and the integrated value of the SFSWWA operator is still a SFV, so we have:

$$SFSWWG\left(\overline{\overline{\mathfrak{D}_1}},\overline{\overline{\mathfrak{D}_2}},\ldots,\overline{\overline{\mathfrak{D}_{ñ}}}\right)=\left(\begin{array}{c}\sqrt{\frac{1}{\Upsilon}\left((1+\Upsilon)\prod_{\tau=1}^{ñ}\left(\frac{\Upsilon\overline{\overline{\mathcal{U}_\tau^2}}+1}{1+\Upsilon}\right)^{\mathrm{w}_\tau}-1\right)},\\ \sqrt{\frac{1+\Upsilon}{\Upsilon}\left(1-\prod_{\tau=1}^{ñ}\left(1-\overline{\overline{\mathcal{V}_\tau^2}}\left(\frac{\Upsilon}{1+\Upsilon}\right)\right)^{\mathrm{w}_\tau}\right)},\\ \sqrt{\frac{1+\Upsilon}{\Upsilon}\left(1-\prod_{\tau=1}^{ñ}\left(1-\overline{\overline{\mathcal{Y}_\tau^2}}\left(\frac{\Upsilon}{1+\Upsilon}\right)\right)^{\mathrm{w}_\tau}\right)}\end{array}\right)$$

Theorem 6: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}}=\left(\bar{\bar{\mathcal{U}}}_\tau,\overline{\overline{\mathcal{V}_\tau}},\overline{\overline{\mathcal{Y}_\tau}}\right),\tau=1,2,\ldots,ñ$, implies that $\overline{\overline{\mathfrak{D}_\tau}}=\bar{\bar{\mathfrak{D}}}$. Then we have:

$$SFSWWA\left(\overline{\overline{\mathfrak{D}_1}},\overline{\overline{\mathfrak{D}_2}},\ldots,\overline{\overline{\mathfrak{D}_{ñ}}}\right)=\bar{\bar{\mathfrak{D}}}$$

Theorem 7: For any two sets of a SFVs $\overline{\overline{\mathfrak{D}_\tau}}=\left(\bar{\bar{\mathcal{U}}}_\tau,\overline{\overline{\mathcal{V}_\tau}},\overline{\overline{\mathcal{Y}_\tau}}\right)$ and $\overline{\overline{\mathfrak{D}'_\tau}}=\left(\bar{\bar{\mathcal{U}}}'_\tau,\overline{\overline{\mathcal{V}'_\tau}},\overline{\overline{\mathcal{Y}'_\tau}}\right),\tau=1,2,\ldots,ñ$. If $\overline{\overline{\mathfrak{D}_\tau}}\leq\overline{\overline{\mathfrak{D}'_\tau}}$, so we have:

$$SFSWWA\left(\overline{\overline{\mathfrak{D}_1}},\overline{\overline{\mathfrak{D}_2}},\ldots,\overline{\overline{\mathfrak{D}_{ñ}}}\right)\leq SFSWWA\left(\overline{\overline{\mathfrak{D}'_1}},\overline{\overline{\mathfrak{D}'_2}},\ldots,\overline{\overline{\mathfrak{D}'_{ñ}}}\right)$$

Theorem 8: Let a series of SFVs $\overline{\overline{\mathfrak{D}_\tau}}=\left(\bar{\bar{\mathcal{U}}}_\tau,\overline{\overline{\mathcal{V}_\tau}},\overline{\overline{\mathcal{Y}_\tau}}\right),\tau=1,2,\ldots,ñ$. If $\overline{\overline{\mathfrak{D}^-}}=\left(\min\left\{\overline{\overline{\mathcal{U}_\tau}}\right\},\max\left\{\overline{\overline{\mathcal{V}_\tau}}\right\},\max\left\{\overline{\overline{\mathcal{Y}_\tau}}\right\}\right)$ and $\overline{\overline{\mathfrak{D}^+}}=\left(\max\left\{\overline{\overline{\mathcal{U}_\tau}}\right\},\min\left\{\overline{\overline{\mathcal{V}_\tau}}\right\},\min\left\{\overline{\overline{\mathcal{Y}_\tau}}\right\}\right)$. Then we have:

$$\overline{\overline{\mathfrak{D}^-}}\leq SFSWWA\left(\overline{\overline{\mathfrak{D}_1}},\overline{\overline{\mathfrak{D}_2}},\ldots,\overline{\overline{\mathfrak{D}_{ñ}}}\right)\leq\overline{\overline{\mathfrak{D}^+}}$$

4. Advanced Decision-Making Methodologies for spherical Fuzzy Information

The MADM problem involves every field of life and different applications under consideration of various types of attribute information. Numerous research scholars have recently developed mathematical approaches for handling complicated real-life applications and numerical examples. To achieve more tangible aggregated outcomes from incomplete and vague information, we have developed an efficient decision algorithm for the MADM problem. To serve this purpose, let a set of n alternative $\{\mathfrak{J}_1,\mathfrak{J}_2,\ldots,\mathfrak{J}_n\}$ and a family of m attributes $\{\mathfrak{M}_1,\mathfrak{M}_2,\ldots,\mathfrak{M}_m\}$ with some specific degree of weight $(\mathfrak{w}_1,\mathfrak{w}_2,\ldots,\mathfrak{w}_m),\mathfrak{w}_j>0$ to each attribute. To demonstrate an ideal solution from the available optimal option, express a decision algorithm of the MADM problem as follows:

Step 1: Construct a decision matrix with information about various attributes corresponding to each alternative or individual.

Step 2: Normalize the standard decision matrix into a modified decision matrix using the following expression:

$$R'=\begin{cases}\left(\bar{\bar{\mathcal{U}}}_{\tau j},\overline{\overline{\mathcal{V}_{\tau j}}},\overline{\overline{\mathcal{Y}_{\tau j}}}\right) & beneficial\ type\\ \left(\overline{\overline{\mathcal{Y}_{\tau j}}},\overline{\overline{\mathcal{V}_{\tau j}}},\bar{\bar{\mathcal{U}}}_{\tau j}\right) & non-beneficial\ type\end{cases}$$

Step 3: Applied derived approaches of the SFSWWA and SFSWWG operators.

Step 4: Compute the score function of each alternative using the Definition of 5.

Step 5: Rank alternatives based on computed score values and the highest score function is a more appropriate optimal option.

Numerical Example

Social media apps have revolutionized communication by providing accessible, real-time platforms for connecting with a global audience. They enable instant messaging, video calls, and multimedia sharing, making interactions more dynamic and engaging. These apps facilitate community building, professional networking, and effective information dissemination. They also support businesses in marketing, customer engagement, and feedback collection. In this numerical example, we evaluated five different social media apps for communication with each other based on particular characteristics and features. Consider there are five social media apps $\{\mathbb{A}_1, \mathbb{A}_2, \mathbb{A}_3, \mathbb{A}_4, \mathbb{A}_5\}$ under consideration based on the following four criteria or attributes information as follows:

i. Customizable Privacy Settings J_1:
ii. Collaboration and Networking J_2:
iii. Content Sharing Method J_3:
iv. Secure Login J_4:

To evaluate different social media apps based on the above-discussed attributes and the latest features of social media apps. Before the aggregation process, the expert assigns some degree to each criterion or attribute as $(0.23, 0.28, 0.35, 0.14)$. The evaluation process for different alternatives under the following steps of the MADM problem.

Step 1: The expert demonstrates information about alternatives based on four attributes or characteristics in Table 1.

Step 2: The discussed case study only one type of attribute information like beneficial type. Therefore, the normalization process is not necessary to normalize the decision matrix.

Table 1
Decision matrix of spherical fuzzy information

Alternatives	$\boldsymbol{J_1}$	$\boldsymbol{J_2}$	$\boldsymbol{J_3}$	$\boldsymbol{J_4}$
$\mathbb{A}_1$	$(0.34, 0.64, 0.23)$	$(0.37, 0.68, 0.61)$	$(0.62, 0.51, 0.48)$	$(0.72, 0.27, 0.63)$
$\mathbb{A}_2$	$(0.56, 0.53, 0.46)$	$(0.48, 0.39, 0.65)$	$(0.59, 0.66, 0.43)$	$(0.64, 0.31, 0.55)$
$\mathbb{A}_3$	$(0.47, 0.63, 0.35)$	$(0.66, 0.38, 0.55)$	$(0.28, 0.76, 0.36)$	$(0.53, 0.41, 0.62)$
$\mathbb{A}_4$	$(0.51, 0.27, 0.69)$	$(0.52, 0.31, 0.61)$	$(0.62, 0.32, 0.68)$	$(0.55, 0.37, 0.66)$
$\mathbb{A}_5$	$(0.63, 0.54, 0.35)$	$(0.32, 0.66, 0.27)$	$(0.43, 0.57, 0.53)$	$(0.44, 0.37, 0.64)$

Step 3: Applied derived aggregation operators, and aggregated results are shown in Table 2.

Table 2
Aggregated decision matrix

Alternatives	SFSWWA	SFSWWG
$\mathbb{A}_1$	$(0.5282, 0.5640, 0.4958)$	$(0.5127, 0.5744, 0.5071)$
$\mathbb{A}_2$	$(0.5629, 0.5159, 0.5198)$	$(0.5607, 0.5275, 0.5263)$
$\mathbb{A}_3$	$(0.4953, 0.5869, 0.4558)$	$(0.4811, 0.6068, 0.4638)$
$\mathbb{A}_4$	$(0.5597, 0.3139, 0.6602)$	$(0.5580, 0.3143, 0.6611)$
$\mathbb{A}_5$	$(0.4638, 0.5644, 0.4454)$	$(0.452, 0.5696\ 0.4564)$

Step 4: Investigated score values corresponding to each alternative listed in Table 3.

Table 3
Score functions of alternatives

Alternatives	SFSWWA	SFSWWG
A_1	-0.0949	-0.1081
A_2	-0.0732	-0.0803
A_3	-0.1023	-0.1173
A_4	-0.0737	-0.0748
A_5	-0.1006	-0.1082

Step 5: Table 4 arranges alternatives based on aggregated score values shown in Table 3.

Table 4
Ranking of alternatives

Aggregation Operators	Ranking and Ordering
SFSWWA	$A_2 \succ A_4 \succ A_1 \succ A_5 \succ A_3$
SFSWWG	$A_4 \succ A_2 \succ A_1 \succ A_5 \succ A_3$

Figure 1 facilitates a clearer understanding of the aggregated results and bahavior of computed results shown in Table 4. From Figure 1, we can easily examine the highest computed results of score functions corresponding to alternatives.

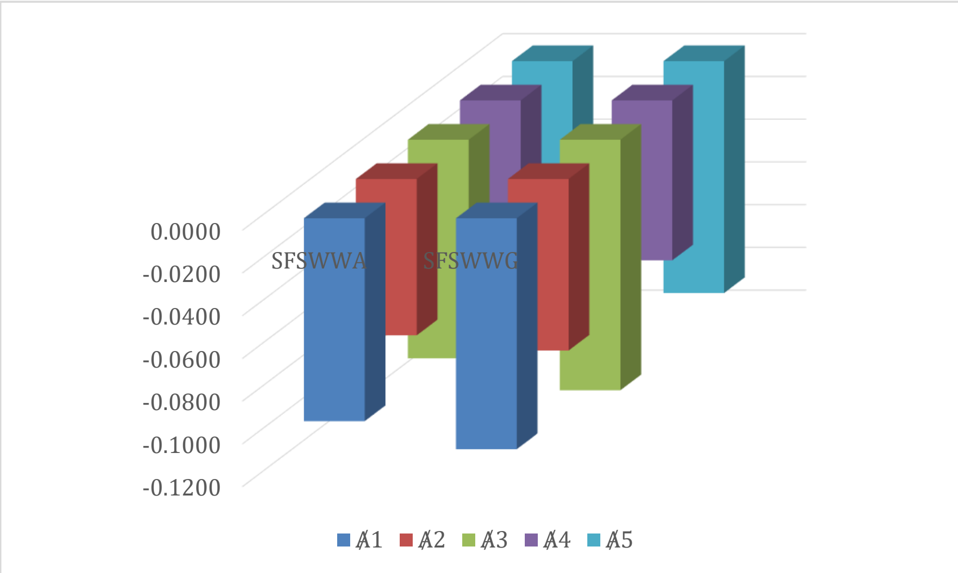

Fig. 1. The geometrical representation of computed score values.

5. Conclusion

This article demonstrated a potent approach to an advanced decision-making technique for the MADM problem under consideration of various characteristics or attributes information. Decision-making is a crucial and complicated task due to incomplete and vague information about real-life applications and numerical examples. An appropriate model of the spherical fuzzy framework is used to eliminate the effect of incomplete information on human opinions. Some feasible operations of Sugeno-Weber t-norms are also described in the light of the spherical fuzzy framework. Motivated by the significance of Sugeno-Weber operations, we constructed a family of Sugeno-Weber aggregation operators under consideration of spherical fuzzy theory. Finally, we applied derived theories on the MADM problem and resolved some real-life applications with the help of numerical examples. After the aggregation process, we examined some appropriate optimal options based on various characteristics or attribute information. Additionally, the geometrical representation demonstrates all aggregated outcomes in a figure of 3-D column. Moreover, we can expand deduced theories in different fuzzy frameworks and mathematical methodologies. The derived aggregation operators can be used to find tangible results from advanced optimization techniques and complicated real-life applications.

Author Contributions

For research articles "Conceptualization, A.H. and K.U.; methodology, A.H., K.U.; software, A.H.; validation, A.H., and K.U.; formal analysis, K.U.; investigation, A.H.; resources, A.H.; data curation, K.U.; writing—original draft preparation, A.H.; writing—review and editing, A.H., K.U.; visualization, K.U.; supervision, K.U.; project administration, K.U.; funding acquisition, A.H. All authors have read and agreed to the published version of the manuscript.

Funding

This research received no external funding.

Data Availability Statement

The data will be available on reasonable request to the corresponding author.

Conflicts of Interest

The authors declare that they have no conflict of interest.

References

[1] Atanassov, K. T. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

[2] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[3] Garg, H., & Kumar, K. (2019). Multiattribute decision making based on power operators for linguistic intuitionistic fuzzy set using set pair analysis. Expert Systems, 36(4), e12428. https://doi.org/10.1111/exsy.12428

[4] Garg, H. (2020). Linguistic Interval-Valued Pythagorean Fuzzy Sets and Their Application to Multiple Attribute Group Decision-making Process. Cognitive Computation, 12(6), 1313–1337. https://doi.org/10.1007/s12559-020-09750-4

[5] Yager, R. R. (2016). Generalized orthopair fuzzy sets. IEEE Transactions on Fuzzy Systems, 25(5), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005

[6] Cuong, B. (2015). Picture fuzzy sets. Journal of Computer Science and Cybernetics, 30. https://doi.org/10.15625/1813-9663/30/4/5032

[7] Yager, R. R. (2013). Pythagorean fuzzy subsets. 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), 57–61. https://doi.org/10.1109/IFSA-NAFIPS.2013.6608375

[8] Cuong, B. C. (2013). Picture fuzzy sets-first results. Part 1, seminar neuro-fuzzy systems with applications. Institute of Mathematics, Hanoi.

[9] Atanassov, K., & Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31(3), 343–349. https://doi.org/10.1016/0165-0114(89)90205-4

[10] Cuong, B. (2015). Picture fuzzy sets. Journal of Computer Science and Cybernetics, 30. https://doi.org/10.15625/1813-9663/30/4/5032

[11] Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. Neural Computing and Applications, 31(11), 7041–7053. https://doi.org/10.1007/s00521-018-3521-2

[12] Xu, Z. (2007). Intuitionistic fuzzy aggregation operators. IEEE Transactions on Fuzzy Systems, 15(6), 1179–1187. https://doi.org/10.1109/TFUZZ.2006.890678

[13] Xu, Z., & Yager, R. R. (2006). Some geometric aggregation operators based on intuitionistic fuzzy sets. International Journal of General Systems, 35(4), 417–433. https://doi.org/10.1080/03081070600574353

[14] Peng, X., & Yang, Y. (2015). Some Results for Pythagorean Fuzzy Sets: Some results for Pythagorean fuzzy sets. International Journal of Intelligent Systems, 30(11), 1133–1160. https://doi.org/10.1002/int.21738

[15] Hussain, A., Ullah, K., Al-Quran, A., & Garg, H. (2023). Some T-spherical fuzzy dombi hamy mean operators and their applications to multi-criteria group decision-making process. Journal of Intelligent & Fuzzy Systems, 45(6), 9621–9641. https://doi.org/10.3233/JIFS-232505

[16] Ren, P., Xu, Z., & Gou, X. (2016). Pythagorean fuzzy TODIM approach to multi-criteria decision making. Applied Soft Computing, 42, 246–259. https://doi.org/10.1016/j.asoc.2015.12.020

[17] Garg, H. (2017a). Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t-norm and t-conorm for multicriteria decision-making process. International Journal of Intelligent Systems, 32(6), 597–630. https://doi.org/10.1002/int.21860

[18] Kaur, G., & Garg, H. (2019). Generalized cubic intuitionistic fuzzy aggregation operators using t-norm operations and their applications to group decision-making process. Arabian Journal for Science and Engineering, 44, 2775–2794. https://doi.org/10.1007/s13369-018-3532-4

[19] Yang, L., & Li, B. (2020). Multiple-valued picture fuzzy linguistic set based on generalized heronian mean operators and their applications in multiple attribute decision making. IEEE Access, 8, 86272–86295. https://doi.org/10.1109/ACCESS.2020.2992434

[20] Liu, P., & Wang, P. (2018). Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. International Journal of Intelligent Systems, 33(2), 259–280. https://doi.org/10.1002/int.21927

[21] Yang, Z., Garg, H., Li, J., Srivastava, G., & Cao, Z. (2021). Investigation of multiple heterogeneous relationships using a q-rung orthopair fuzzy multi-criteria decision algorithm. Neural Computing and Applications, 33(17), 10771–10786. https://doi.org/10.1007/s00521-020-05003-5

[22] Hussain, A., Ullah, K., Alshahrani, M. N., Yang, M.-S., & Pamucar, D. (2022). Novel Aczel–Alsina Operators for Pythagorean Fuzzy Sets with Application in Multi-Attribute Decision Making. Symmetry, 14(5), 940. https://doi.org/10.3390/sym14050940

[23] Ali, Z., Mahmood, T., & Yang, M.-S. (2020). TOPSIS Method Based on Complex Spherical Fuzzy Sets with Bonferroni Mean Operators. Mathematics, 8(10), Article 10. https://doi.org/10.3390/math8101739

[24] Dong, H., Ali, Z., Mahmood, T., & Liu, P. (2023). Power aggregation operators based on hamacher t-norm and t-conorm for complex intuitionistic fuzzy information and their application in decision-making problems. Journal of Intelligent & Fuzzy Systems, Preprint, 1–21. https://doi.org/10.3233/JIFS-230323

[25] Hussain, A., Bari, M., & Javed, W. (2022). Performance of the Multi Attributed Decision-Making Process with Interval-Valued Spherical Fuzzy Dombi Aggregation Operators. Journal of Innovative Research in Mathematical and Computational Sciences, 1(1), 1-32.

[26] Hussain, A., Latif, S., & Ullah, K. (2022). A Novel Approach of Picture Fuzzy Sets with Unknown Degree of Weights based on Schweizer-Sklar Aggregation Operators. Journal of Innovative Research in Mathematical and Computational Sciences, 1(2), 18-39.

[27] Garg, H. (2017b). Some Picture Fuzzy Aggregation Operators and Their Applications to Multicriteria Decision-Making. Arabian Journal for Science and Engineering, 42(12), 5275–5290. https://doi.org/10.1007/s13369-017-2625-9

[28] Wang, C., Zhou, X., Tu, H., & Tao, S. (2017). Some geometric aggregation operators based on picture fuzzy sets and their application in multiple attribute decision making. Italian journal of pure and applied mathematics, 37, 477–492.

[29] Ullah, K., Hassan, N., Mahmood, T., Jan, N., & Hassan, M. (2019). Evaluation of Investment Policy Based on Multi-Attribute Decision-Making Using Interval Valued T-Spherical Fuzzy Aggregation Operators. Symmetry, *11*(3), 357 https://doi.org/10.3390/sym11030357

[30] Hussain, A., Ullah, K., Pamucar, D., Haleemzai, I., & Tatić, D. (2023). Assessment of Solar Panel Using Multiattribute Decision-Making Approach Based on Intuitionistic Fuzzy Aczel Alsina Heronian Mean Operator. International Journal of Intelligent Systems, 2023, e6268613. https://doi.org/10.1155/2023/6268613

[31] Hussain, A., Ullah, K., Senapati, T., & Moslem, S. (2023). A robust decision-making approach for supplier selection using complex picture fuzzy information involving prioritization of attributes. IEEE Access, 11, 91807–91830. https://doi.org/10.1109/ACCESS.2023.3308030

[32] Yager, R. R. (2001). The power average operator. IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans, 31(6), 724–731. https://doi.org/10.1109/3468.983429

[33] Xu, Z., & Yager, R. R. (2010). Intuitionistic fuzzy Bonferroni means. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 41(2), 568–578. https://doi.org/10.1109/TSMCB.2010.2072918

[34] Wei, G., Zhao, X., Wang, H., & Lin, R. (2013). Fuzzy power aggregation operators and their application to multiple attribute group decision making. Technological and Economic Development of Economy, 19(3), 377–396. https://doi.org/10.3846/20294913.2013.821684

[35] Wei, G., & Lu, M. (2018). Pythagorean fuzzy power aggregation operators in multiple attribute decision making. International Journal of Intelligent Systems, 33(1), 169–186. https://doi.org/10.1002/int.21946

[36] Xu, Z. (2011). Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators. Knowledge-Based Systems, 24(6), 749–760. https://doi.org/10.1016/j.knosys.2011.01.011

[37] Sugeno, M. (1974). Theory of fuzzy integrals and its applications. Doctoral Thesis, Tokyo Institute of Technology.

[38] Weber, S. (1983). A general concept of fuzzy connectives, negations and implications based on t-norms and t-conorms. Fuzzy Sets and Systems, 11(1–3), 115–134. https://doi.org/10.1016/S0165-0114(83)80073-6

[39] Ashraf, S., Iqbal, W., Ahmad, S., & Khan, F. (2023). Circular Spherical Fuzzy Sugeno Weber Aggregation Operators: A Novel Uncertain Approach for Adaption a Programming Language for Social Media Platform. IEEE Access. https://doi.org/10.1109/ACCESS.2023.3329242

[40] Hwang, C.-M., Yang, M.-S., Hung, W.-L., & Lee, M.-G. (2012). A similarity measure of intuitionistic fuzzy sets based on the Sugeno integral with its application to pattern recognition. Information Sciences, 189, 93–109. https://doi.org/10.1016/j.ins.2011.11.029

[41] Pamucar, D., Lazarević, D., Dobrodolac, M., Simic, V., & Görçün, Ö. F. (2024). Prioritization of crowdsourcing models for last-mile delivery using fuzzy Sugeno–Weber framework. Engineering Applications of Artificial Intelligence, 128, 107414. https://doi.org/10.1016/j.engappai.2023.107414

[42] Sarkar, A., Senapati, T., Jin, L., Mesiar, R., Biswas, A., & Yager, R. R. (2023). Sugeno–Weber Triangular Norm-Based Aggregation Operators Under T-Spherical Fuzzy Hypersoft Context. Information Sciences, 119305. https://doi.org/10.1016/j.ins.2023.119305

[43] Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. Neural Computing and Applications, 31(11), 7041–7053. https://doi.org/10.1007/s00521-018-3521-2