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Optimum Production Lot Size for a Perishable Product Under Exponential Demand with Partial Backordering and Rework

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ABSTRACT

The management of inventory significantly impacts an organization's profitability. Numerous researchers have developed the production model tailored for decaying items to boost sales and minimize inventory, thereby increasing turnover ratios. This study focuses on the EPQ model for reworkable items under exponential combined with inventory dynamics. The primary objective is to optimize the production plan to manage maintenance costs effectively, ensuring optimal Leveraging human and technological resources. The research determines the optimal cycle length, backordered quantities, and production quantities to minimize overall costs. Specifically, penalties are incurred for excessive production times during rework and non-rework phases. This model finds applications in industries such as printed circuit board assembly in semiconductor manufacturing, metal parts, and plastic molding. The study concludes with numerical examples comparing total costs under the proposed model, supplemented by sensitivity analyses that explore the impacts of parameter variations on decision-making.

1. Introduction

Inventory control involves managing the investment in materials and parts kept in stock within predefined limits, as established by management's inventory policy. This paper explores the Production model that incorporates both manufacturing and repair processes. In practical production settings, defects can arise during manufacturing, requiring rework to achieve perfection. Rework is essential in reverse supply chain management and eco friendly initiatives, as it can diminish manufacturing costs.

To optimize the system's total cost, a robust iterative solution method is devised. Managing combined demand in inventory poses challenges not only in planning but also in executing and physically controlling requirements. For instance, consider the case of an electrical motor

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demanded independently by its manufacturer but as a dependent demand (a purchased subassembly) for a refrigerator manufacturer. This iterative approach aims to minimize costs and improve efficiency in inventory management amidst complex production and demand dynamics. Increasing the number of production setups can automatically reduce total production costs, along with decreasing penalty maintenance costs, price discounts, and shortages. Significant reduction in rework can be achieved through meticulous project planning, supervision, and rigorous implementation of quality assurance measures. Such efforts are crucial for reducing overall project costs, thereby increasing profitability for producers and lowering final costs for customers.

This research study primarily focuses on defective items, rework processes, and pricing penalties. Consequently, it reviews pertinent literature in these areas. With the globalization of business companies, these strategies are essential for achieving operational goals by optimizing resource utilization and minimizing costs.

Additionally, it explores how these influences are related to production operational efficiency indicators and ecological factors by Adauto *et al.*, [1]. an explicit solution is obtained to maximize the expected average net profit for both consolidate and decentralized scenarios was explained in Barman *et al.*, [2]

Dari and Sani [3] explained an Economic Production Quantity (EPQ) model for items with delayed deterioration. In this model, the demand before deterioration begins is assumed to follow a time-dependent quadratic pattern, and shortages are permitted and partially backordered. Gwanda *et al.*, [4] consider the rate of amelioration as a time-dependent function and assume that The total manufacturing expenses are inversely related to the rate of demand. The model is used to demonstrate optimization at three different time points. Ivanov [5] demonstrated statistical demand forecasting methods and inventory control policies, showing how to manage demand and lead time uncertainty. Kumar and Raput.,[6] emphasizes green supply chain models. Kuppulakshmi *et al.,[*7] her paper examines the manufacturing model for reworkable items under three scenarios: The main focus of the study is on maintenance costs. Kuppulakshmi *et al*.,[8] examined the fuzzy economic model for the manufacturing inventory system with an imperfect production process, incorporating rework. During the pandemic, it became evident that products accumulated without being sold, leading to increased maintenance costs.

Lalremruati& Aditi Khanna., [9] flourished the model explained a flexible production rate based on working labor, energy and mechanism, aiming to meet demand while reducing waste. The real-world issues are taken into account while modeling an inventory for a cleaner production system by Pankaj Bhatnagar *et al*.,[10]. Peeters & van Ooijen [11] demonstrate how various situations by classifying pertinent material using a unique taxonomy. Furthermore, a summary of the modeling strategies and approaches applied in these publications is given. The optimality requirements of an unconstrained interval optimization problem are established in parametric form by Rahman [12].

An unconstrained interval optimization problem's maximizer and minimize are defined with the aid of the suggested order relation.With the aim of minimizing overall cost, Rani et al. [13] developed an understandable integrated model for the manufacturing and refurbishment cycle, assuming that the parameters for demand, deterioration, collection, and cannibalization are exactly known. A functioning model for non-instantaneous deteriorating objects was discussed by Shah & Naik [14]. They included the learning effect on different expenses and the investment in preservation technologies to decrease the deterioration rate.

A production model with reverse logistics is presented by Subhas Kumar et al. [15] to find faulty products. The concept makes use of green technologies and incorporates learning via production. Furthermore, after being delivered to customers, some of the completed goods are remanufactured. After screening, the refurbished goods are delivered to markets. Taleizadeh et al., [16] established and studied an inventory model including interruption in process, scrap, and rework. An inventory control model described by Taleizadeh et al.,[17] this study determines the best order and shortage amounts of a perishable item when the supplier offers an exclusive discount.

Taleizadeh et al. [18] created a three-level supply chain consisting of several non-competing suppliers, a single manufacturer, and multiple restrictive covenants retailers for different items with repair processes in integrated and non-integrated structures. Taleizadeh et al., [19] identify this new form of production-inventory model as one of the characteristics that make inventory management more realistic and practical for managers. Yao*et al.*, [20] demonstrated the constant demand rate, whereas in marketing and pricing, it is determined by the trade value.

2. Methodology

This work examines an Economic Production Quantity (EPQ) model that integrates penalty preservation costs, rework processes, discount prices and shortages. Demand is analyzed using both exponential and linear functions across three distinct scenarios. The total cycle time is divided into four key parameters: production, non-production, rework, and non-rework. During the rework phase, maintenance costs are considered penalty maintenance costs. Increasing the number of production cycles reduces total time and overall production costs. Rework begins after the production phase is completed. Figure 1 depicts the EPQ model with rework. In this system, items are inspected for quality before meeting customer demand, guaranteeing only items that meet quality criteria are stored and sold. The remanufacturing process is particularly relevant in industries such as manufacturing leather and electronics.

Production begins at time $\tau = 0$ with the inventory level initially at zero. Manufacturing Ratio is denoted by P and the rate of demand by D. In production model with a revamp process is that the manufacturing rate, minus imperfect products, must always comply with or exceed demand. Manufacturing and supply begin concurrently, and inventory reaches its peak level $\mathrm{I}_{\max R}$ at period $\tau = T_1$. After this point, production ceases, and the available inventory gradually diminishes due to ongoing demand and losses from deterioration. This process is repeated over n cycles, with defective goods continuously sent for rework. It is assumed that all defective products can be repaired until no scrap remains at the end of the rework phase.Shortages are permitted, and any unsatisfied demands are backordered. During the rework period, produced items cannot fulfill current demand, leading to backlogging. As the selling price fluctuates, the holding cost per unit decreases. Repair begins after a particular production time, and the repair rate differs from the production rate. During the rework phase, inventory reaches its maximum.

Basic notations and calculations are adapted from the Kuppulakshmi*et al.*,[8] , and this includes considering rework in non-production inventory with exponential demand.

$$\frac{dI_{p_1}(\tau_1)}{d\tau_1} + \omega I_{p_1}(\tau_1) = gp - D, \quad 0 \le \tau_1 \le T_1 \tag{1}$$

$I_{p_1}(0) = 0$

period $[0, T_2]$ is given below

$$\frac{dI_{p_2}(\tau_2)}{d\tau_2} + \omega I_{p_2}(\tau_2) = -D \quad 0 \le \tau_2 \le T_2 \tag{2}$$

When $I_{p_2}(0) = 0$

The stock level in the revamp time $[0, T_3]$ is given as

$$\frac{dI_{p_3}(\tau_3)}{d\tau_3} + \omega I_{p_3}(\tau_3) = p_r - D \quad 0 \le \tau_3 \le T_3 \tag{3}$$

with the $I_{p_3}(0) = 0$

The stock range in non revamp $[0, T_4]$ is given below

$$\frac{dI_{p_4}(\tau_4)}{d\tau_4} + \omega I_{p_4}(\tau_4) = -D \quad 0 \le \tau_4 \le T_4 \tag{4}$$

with the condition that $I_{p_4}(0) = 0$

$$D = \alpha e^{\beta\tau} + \beta I(\tau) \tag{5}$$

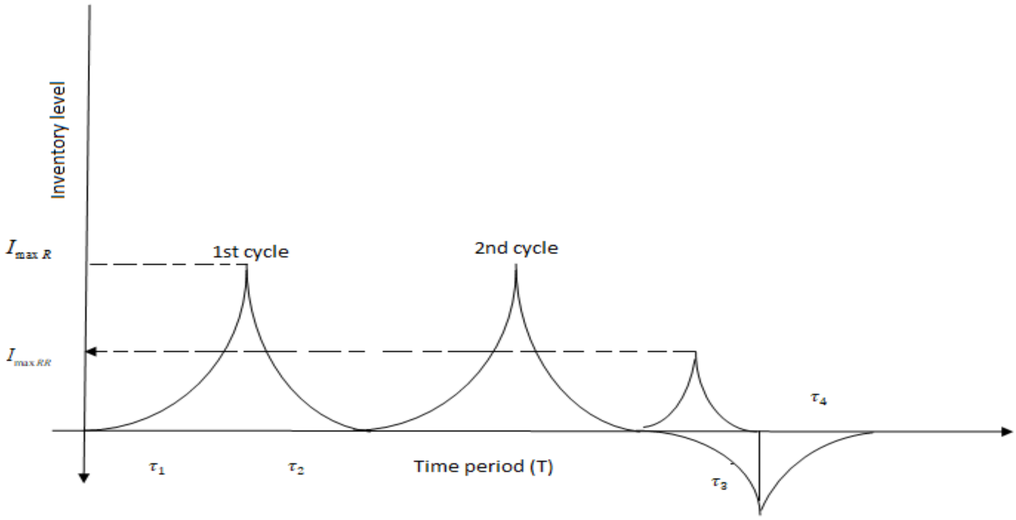

Fig. 1. Production Inventory Level with Rework

By solving equation (1),(2),(3) and (4) using initial condition, the operational inventory level

$$I_{\tau p_1}(\tau_1) = \frac{gp-\alpha}{2}{T_1}^2 \tag{6}$$

$$I_{\tau p_2}(\tau_2) = \frac{\alpha}{2}{T_2}^2 \tag{7}$$

$$T_2 = \left(\frac{gp}{\alpha}\right)\left[T_1 - \frac{(\omega+\beta){T_1}^2}{2}\right] - \left[T_1 - \frac{\omega{T_1}^2}{2}\right] \tag{8}$$

$$I_{\tau p_3}(\tau_3) = \frac{p_r-\alpha}{2}{T_3}^2 \tag{9}$$

$$I_{\tau p_4}(\tau_4) = \frac{\alpha}{2}{T_4}^2 \tag{10}$$

$$T_4 = \frac{p_r-\alpha}{\alpha}\left[T_3 - \frac{(\omega+\beta){T_3}^2}{2}\right] \tag{11}$$

$$I_{\tau r1}\ (\tau_{r1}) = \frac{(1-g)P}{2}{T_1}^2 \tag{12}$$

Initially the recoverable items are denoted by I_{maxR}

$$I_{maxR} = (1-g)P(T_1 - \frac{\omega{T_1}^2}{2}) \tag{13}$$

$$I_{\tau r2}\ (0) = I_{maxR} \tag{14}$$

The total inventory level of recoverable items in the non production period is

$$I_{\tau r2}\ (\tau_{r2}) = \int_{\tau_{r2}=0}^{(n-1)T_1+nT_2} I_{maxR} e^{-\omega\tau_{r2}}\, d\tau_{r2} \tag{15}$$

$$I_{V1} = \sum_{s=1}^{n} I_{maxR}\left[\left((s-1)T_1 + sT_2\right) - \frac{\omega\left((s-1)T_1+sT_2\right)^2}{2}\right] \tag{16}$$

At the end of one production cycle the inventory level attains maximum

$$I_{maxRR} = \sum_{s=1}^{n} I_{maxR} \begin{bmatrix} 1-\omega\big((s-1)T_1+sT_2\big)+ \\ \frac{\omega^2\left((s-1)T_1+sT_2\right)^2}{2} \end{bmatrix} \tag{17}$$

The total recoverable items are

$$I_{\tau r3} = \frac{P_{rT_3}{}^2}{2} \tag{18}$$

Equation (18) can be written as

$$I_{maxRR} = \frac{P_r}{\omega}\left(e^{\omega T_3}-1\right) \tag{19}$$

$$T_3 = \frac{I_{maxRR}}{P_r} \tag{20}$$

$$TOI = nI_{\tau p_1} + nI_{\tau p_2} + I_{\tau p_1} + I_{\tau p_1} \tag{21}$$

$$TRI = nI_{\tau r1} + I_{V1} + I_{\tau r3} \tag{22}$$

$$\theta_T = [ngPT_1 + P_rT_3] - [n(\alpha+\beta I_{\tau p_1})T_1 + n(\alpha+\beta I_{\tau p_2})T_2 + (\alpha+\beta I_{\tau p_3})T_3 + (\alpha+\beta I_{\tau p_4})T_4 \tag{23}$$

$$TC_P = (nA_P + H_P(nI_{\tau r1}) + \omega T_1[ngp - n(\alpha+\beta I_{\tau p_1})])/nT_1 \tag{24}$$

$$TCn_P = (nA_P + H_P(I_{V1}) + \omega T_2[ngp - n(\alpha+\beta I_{\tau p_2})])/nT_2 \tag{25}$$

$$PMC_1 = \pi(T-T_3)P_r \text{Where} \pi = 3.141 \tag{26}$$

$$PD_1 = \frac{kr}{T_3}\int_{\mu}^{T_3} D d\tau_3 \tag{27}$$

$$PD_1 = \frac{kDr(T_3-\mu)}{T_3} \tag{28}$$

$$SHC_r = c\int_0^{T_3} -D d\tau_3 \tag{29}$$

$$SHC_r = -cDT_3 \tag{30}$$

$$TC_r = (A_r + H_r(I_{\tau r3}) + \omega T_3[P_r - n(\alpha+\beta I_{\tau p_3})] + SHC_r + PMC_1 + PD_1)/T_3 \tag{31}$$

$$PMC_2 = \pi(T-T_3)P_r \qquad \text{where } \pi = 3.141 \tag{32}$$

$$SHC_{nr} = c\int_0^{T_4} -D d\tau_4 \tag{33}$$

$$SHC_{nr} = -cDT_4 \tag{34}$$

$$TC_{nr} = (A_r + H_r(I_{\tau r3}) + \omega T_4[P_{nr} - n(\alpha+\beta I_{\tau p_4})] + SHC_{nr} + PMC_2)/T_4 \tag{35}$$

$$\text{TC} = TC_P + TCn_P + TC_r + TC_{nr} \tag{36}$$

$$\frac{\partial TC_P(n,T_1)}{\partial T_1} = 0$$

The optimal value for T_1 is obtained by iterative method. MATLAB 2014 is used to find the experimental result of Equ.(24) to (36).

3. Numerical illustration

Considering the inventory system with the parameters as α = 102, β = 0.5, P = \$1000/unit, $P_r = \$600$, D = \$600, g = 0.94, $A_P = \$6/unit$, $A_r = \$1/unit$, $H_P = \$3/unit$, k = 5, r = 0.1, μ = 0.01, c = 0.15. By iterating the values of n, the optimal solution is found as a nearby value of n, the time and the total cost as T_1=0.27 and TC=\$246730.

4. Sensitivity analysis

When number of cycle's increases, the total cost of the manufacturing process also increases for the three cases, as shown in Table 1 and the optimal total cost obtained in Rework is shown in

Figure 2. Sensitivity analysis for different parameters like $A_p, H_p, P_r, A_r, C, r, D$ and $P$is analysed with the three cases.

Table 1
Total cost variation with the number of productions cycles and production time

n	T_1	Rework case
1	0.27	246730
2	0.2430	138600
3	0.2160	100890
4	0.1890	81000
5	0.1620	68319
6	0.1350	59300
7	0.1080	52432
8	0.0810	47014
9	0.0540	42834
10	0.0270	40127

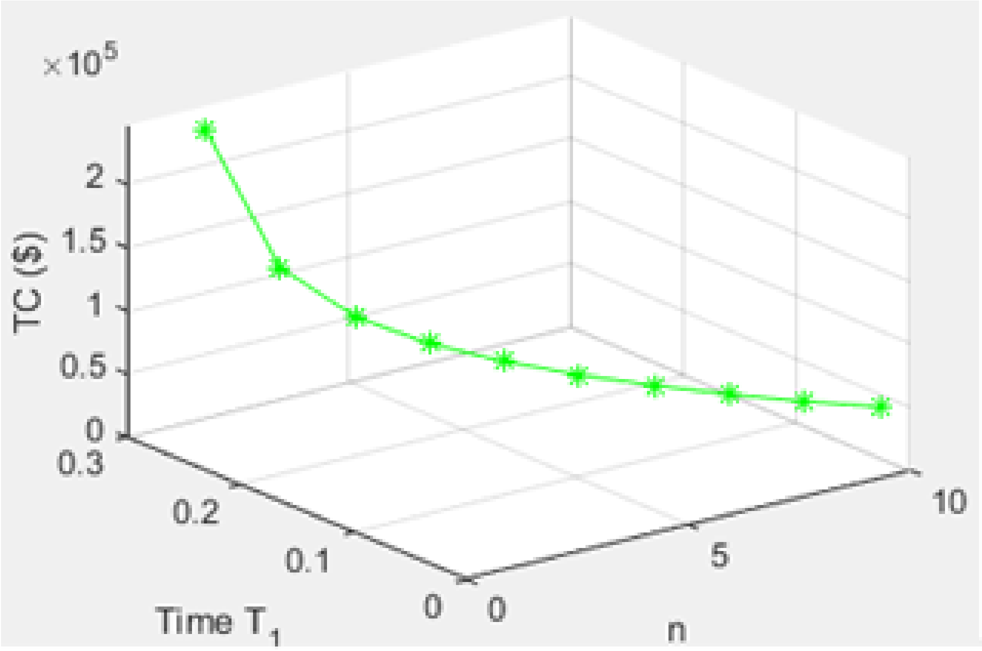

Fig. 2. Optimal total cost variation with the production run time and number of cycles

When the production setup cost varies from -30% to +30%, the total cost correspondingly increases. Likewise, increasing the production rate (P) by -30% to +30% results in a higher total cost. An increase in the demand rate (D) within the same range also leads to a rise in total cost. Adjusting the operational holding cost by -30% to +30% results in an increase in the total cost. Similarly, an increase in the rework process rate within this range leads to a higher total cost. Similarly, increasing the rework production setup cost within -30% to +30% raises the total cost. An increase in the price discount (r) also causes the total cost to rise when adjusted by -30% to +30%. Conversely, a decrease in the shortage cost (c) within this range results in a reduction in total cost is shown in Table 2.

Table 2
Price discount and penalty maintenance cost variations

n	$PD1(\$)$	$PMC1(\$)$	$PMC2(\$)$
1	168.0	4536	4371
2	222.0	4278	4000
3	238.9	3966	3613
4	246.3	3612	3211
5	249.5	3221	2795
6	249.7	2795	2367
7	247.3	2333	1925
8	240.8	1834	1470
9	225.6	1290	999
10	176.5	687	511

5. Conclusions

The analysis and optimization of cost elements that have a major impact on production efficiency are the main goals of the study. These consist of the costs associated with reworkable items, disposal, and penalty missed sales. The goal of the research is to find ways to reduce costs related to defective goods, scrap items, and missed sales opportunities by include these costs in the analysis. Any organization's capacity to maximize earnings and profitability depends heavily on its pricing strategy. As a result, the study looks at overall production costs as a factor in revenue and profit margins, emphasizing how crucial strategic pricing choices are to meeting production planning goals.When Compared with Kuppulakshmi*et al.*,[8] this research achieves a shorter production time of 0.24 than 2.4978.

According to the survey, several researchers have evaluated the EPQ model considering faulty items and screening issues, with a few incorporating the repair/rework option for scrap items. Additionally, exponential demand has been utilized to develop an EPQ model with imperfect items for unknown parameters. However, there are only a few studies on the EPQ model with deteriorated items that apply restoration techniques such as repair options and backlogging. This research aims to address this specific gap.

Author Contributions

Conceptualization and methodology were handled by V. K. Validation, formal analysis, and investigation were conducted by C.S. Review, editing, and visualization were managed by D. N.

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Data Availability Statement

This study did not report any data

Conflicts of Interest

The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results

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