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Analysis of Complex Planetary Gearboxes

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ABSTRACT

The ever-increasing demands for vehicle emission control have resulted in an expansion of planetary gearbox applications in road vehicles, due to their possibility to change the transmission ratio under load in synchronism with engine operation. Modern boxes are designed to extract the maximum number of transmission ratios as possible from the least possible number of simple component planetary gear trains (PGTs). The application demands dictate whether the box will be built with the maximum number of transmission ratios, or built for ruggedness and reliability. Component interconnections are designed to avoid power circulation, hollow shafts, or complex planet carrier arrangements when possible. The subject of this paper are multi-speed complex planetary gearboxes having at least two interconnected simple component PGTs controlled by brakes and clutches. Several variants of complex PGTs are examined together with the placement of brakes and clutches on their external shafts, and their transmission ratio functions are derived. The kinematics of multi speed gear trains are analysed as combinations of two or more two-speed gear trains. Several gearbox layouts are analysed, and their transmission ratio functions provided. Finally, an overview of the procedure for the calculation of creation of multi-speed gear trains is provided.

1. Introduction

Internal combustion engine (ICE) powered vehicles have been in common use for road and off-road vehicle applications since the late 19th century. As an ICE is unable to self-start from standstill, it must have a means to decouple from the wheels and a transmission to match the speed and torque output of the engine to the demands of the powered wheels [1]. The transmission ratio must also be appropriately split between the final drive and the gearbox, which must provide an adequate number of transmission ratios to enable the whole power range of the engine to be used. The transmission

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must also be built with a high internal efficiency, it must be robust, low maintenance, and insensitive to climatic extremes [2,3].

Until the early 2000s, automatic transmissions were restricted to high-powered luxury vehicles, and mechanical transmissions were widespread as they are robust, simple to manufacture and maintain, and can be built with as many gear ratios as required [4]. The main downside of a manual box is the relatively high skill required to operate a manual transmission, especially when dealing with a heavy commercial vehicle, and the requirement of a clutch used to disconnect the engine from the gearbox while changing gears and to “slip” on starting from standstill. This results in wear, and it requires the gearbox to have a very large transmission ratio for the bottom gear and the reverse gear as both must be able to move the vehicle from standstill with the clutch slipping, and experience has shown that only a quarter of the normal engine torque is available for rolling off. This has been well known since the introduction of the Ford Model T, which used a planetary gear train (PGT) connected to the engine through a wet plate clutch. The PGT was kinematically similar to the modern Ravigneaux gearset, providing two forward gears and one reverse gear, selected by depressing a pedal.

Several designs of manual planetary-based boxes followed, usually developed for or derived from military applications. However, the first true automatic gearboxes, able to change gears without reducing or interrupting engine power output, appeared in the early 1930s. These boxes were either built as single or two-stage torque converters with a simple planetary stage for reverse and low gear, or with three or four forward gears derived by a planetary gear train mated to a fluid coupling. All those boxes exhibited low fuel efficiency due to constant slip between the input and output hydrodynamic elements. Efforts were made to improve efficiency by connecting some transmission elements directly to the engine flywheel. On the other the input fluid stage meant that the first gear could be built with a smaller transmission ratio, especially when combined with a torque converter.

Automatics following modern design principles will appear from the 1960s onwards with the widespread adoption of the Simpson gear train and the development of the three-element torque converter equipped with a lock-up clutch. Further development from this point onwards was in the form of adding overdrive and/or low gear trains to existing gearsets and the addition of electronic shifting controls.

Contemporary planetary gearbox design philosophy has evolved in two directions. The first direction is to extract the greatest possible number of gear ratios from a gearset by adding brakes and clutches to interconnect elements, requiring elaborate bearing solutions [5], and is most common for passenger car transmissions. The other design direction, meant for commercial vehicles and buses, prioritizes rugged and reliable design, and prefers to add component geartrains to achieve extra gear ratios. The designs avoid power circulation and reduce the use of hollow shafts to a minimum, especially for sun gears.

Current designs of PGT based gearboxes use a torque converter input stage and can withstand power inputs of about 600 kW, the actual limit being the centrifugal loads inside the gearbox. Modern planetary gearboxes essentially use combinations of two, three, and four-carrier gear trains controlled by clutches, with a single-carrier stage usually used either as a low gear or overdrive stage [6–12]. Individual gear ratios usually work in two-carrier and three-carrier mode, with four-carrier mode being used for a single ratio.

It is well known that the application of PGT based gearboxes provides considerable advantages in comparison to conventional boxes, with expanded possibilities for application [13,14].

The research performed for the purposes of this article was based on two-carrier PGTs with four external shafts composed of two PGTs of the basic type [15,16], extended to more component PGTs as required. An overview of the internal structure of the researched gear trains is provided, and all schemes and layout variants have been systematized. A software program for numerical simulation and calculation of PGT parameters was developed to determine the internal workings and the most important basic parameters of the component gear trains and the whole gear train, due to the large number of permutations involved.

The acceptable transmission solutions analysed in this paper were generated using this specially developed computer program. The calculation procedure and the application of the rule of torques have been demonstrated on some gear train examples and specific design solutions have been pointed out.

2. The Planetary or Epicyclic Gear Train

Shifting capabilities may be added to a compound planetary gear train by means of conveniently placed brakes and clutches. For example, two simple PGTs may be used to obtain 6-12 transmission ratios from just by using brakes to lock elements down and clutches to make or break interconnections [17–21]. In short words, the shifting capabilities of each layout must be explored, and tools for the selection of the most appropriate layout must be available. The basic component gear train for all research in this paper is the 2k-h, type A according to Kudryavtsev, 'AI' according to Arnaudov and Karaivanov, or 1AI according to the German classification system. This simple PGT can be represented by a Wolf-Arnaudov symbol, which marks the sun shaft 1 with a thin line, the ring gear shaft 3 with a thick line, and the planet carrier shaft S by two parallel lines (Figure 1).

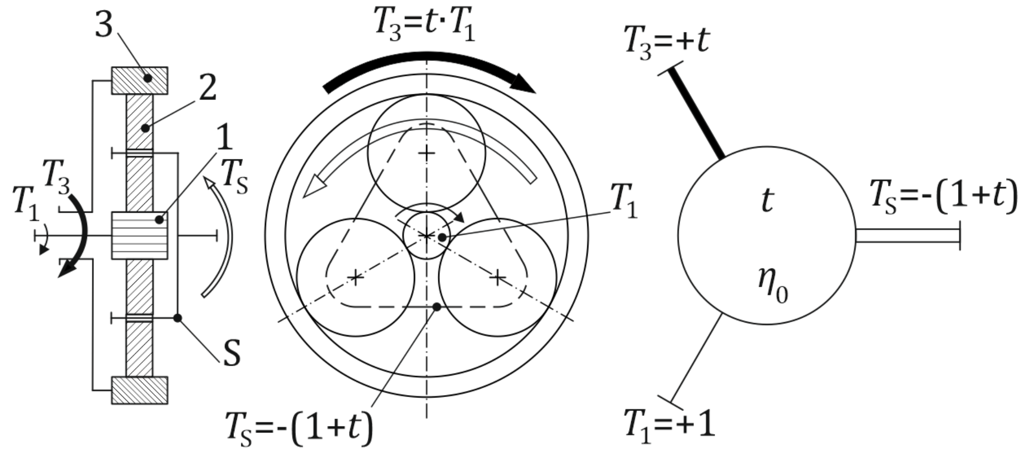

Fig. 1. The most commonly used simple planetary gear train, type 2k-h, variant A

Equations (1-3) are used to form the relationships between the torques acting on the shafts of a simple PGT. The shaft torques are given as functions of the ideal torque ratios (2) which are the basic value when calculating the transmission ratios of complex planetary gearboxes. It should be noted that the equations in this article, unless explicitly stated, do not take efficiency into consideration.

$$\eta_0 = \eta_{13(S)} = \eta_{31(S)} = 1 \tag{1}$$

The ideal torque ratio is defined by (2):

$$t = \frac{T_3}{T_1} = \frac{T_{D\max}}{T_{D\min}} = \left|\frac{z_3}{z_1}\right| > +1 \tag{2}$$

The relations between the torques acting on the shafts of a simple PGT are defined by (3):

$$T_1 : T_3 : T_S = T_{D\min} : T_{D\max} : T_\Sigma = T_1 : t \cdot T_1 : -(1+t) \cdot T_1 = +1 : +t : -(1+t) \tag{3}$$

The following also applies (4):

$$T_1 \equiv T_{D\min} < T_3 \equiv T_{D\max} < \left|T_S\right| \equiv \left|T_\Sigma\right| \tag{4}$$

3. Introduction to Two-carrier Planetary Gear Trains

The characteristics of two-carrier PGTs with two connecting and four external shafts have been extensively analysed in [22,23]. The brakes are placed on two of the external shafts, while the remaining two shafts are used to connect the power source and the load. The reactive members are determined by brake activation, changing both the direction of the power flow and the transmission ratio of the gear train.

These PGTs are composed of two interconnected simple component gear trains as previously shown in Figure 1, as such component gears offer the most advantages in actual use. Analysis has shown that these PGTs can be used for a broad spectrum of combinations of transmission ratios and can be used in a wide range of applications, ranging from machine tools to railway propulsion.

The basic layout of a two-carrier PGT with two connecting and four external shafts may be seen in Figure 2.

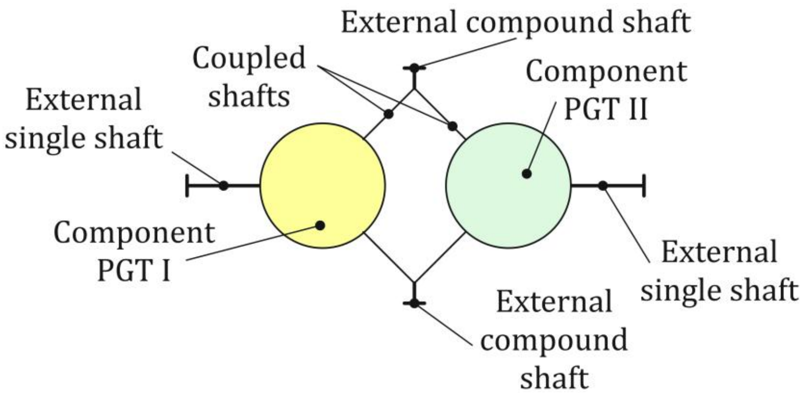

Fig. 2. Schematic layout of two-carrier planetary gear train

The possible layouts for two-carrier PGTs with two connecting and four external shafts are displayed in Figure 3a, while the layout variants according to the convention in [14] are listed in Figure 3b. The locations of brakes Br1 and Br2 are marked, and any variant can be applied to any layout, while the article uses a layout variant nomenclature based on cardinal points.

For example, S13NS(W/E) means that the PGT is built as layout S13, that power input is on the northern external shaft N, and that power output is through the southern external shaft S. (W/E) means that the western and eastern external shafts W and E have brakes mounted on them. Braking shaft W gives layout variant S13NS(W), while braking shaft E gives S13NS(E). The remaining shaft rotates freely and transmits no torque.

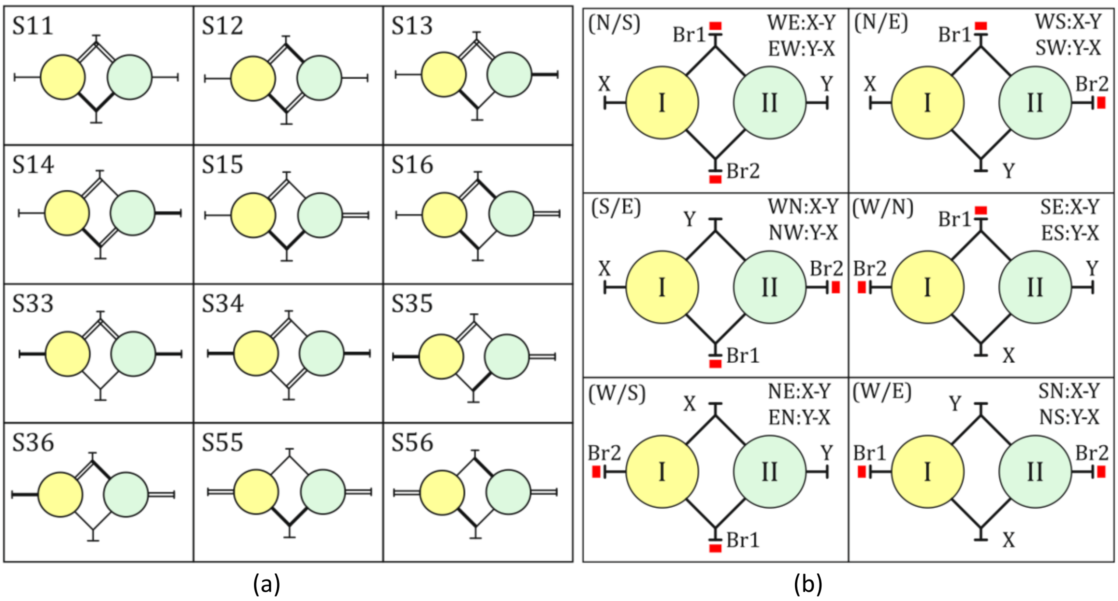

Fig. 3. Possible (a) layouts of two-carrier PGTs with four external shafts, (b) layout variants two-carrier PGTs

Figure 4 displays the kinematic scheme of the S12WS(N/E) planetary gearset. The letters A and B in are used to display the energy flow and therefore denote the layout variant. This gearset will be subject to structural analysis, and the results of displayed in Figure 5.

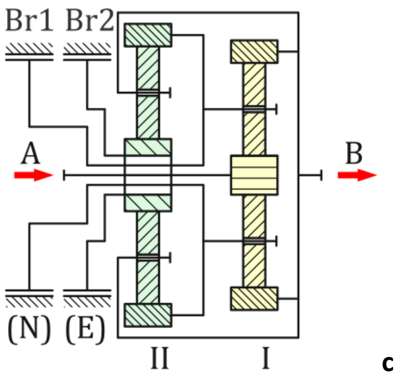

Fig. 4. Kinematic scheme of the S12WS(N/E) PGT

Operation of the gearset with brake Br1 (N) on as S12WS(N) is shown on the left, while the image on the right shows the operation of the gearset with brake Br2 (E) on as S12WS(E). The torque ratios have been determined for all shafts, while the transmission ratios i_{Br1} and i_{Br2} have been calculated as a function of the ideal torque ratios of the component PGTs t_{I} and t_{II}.

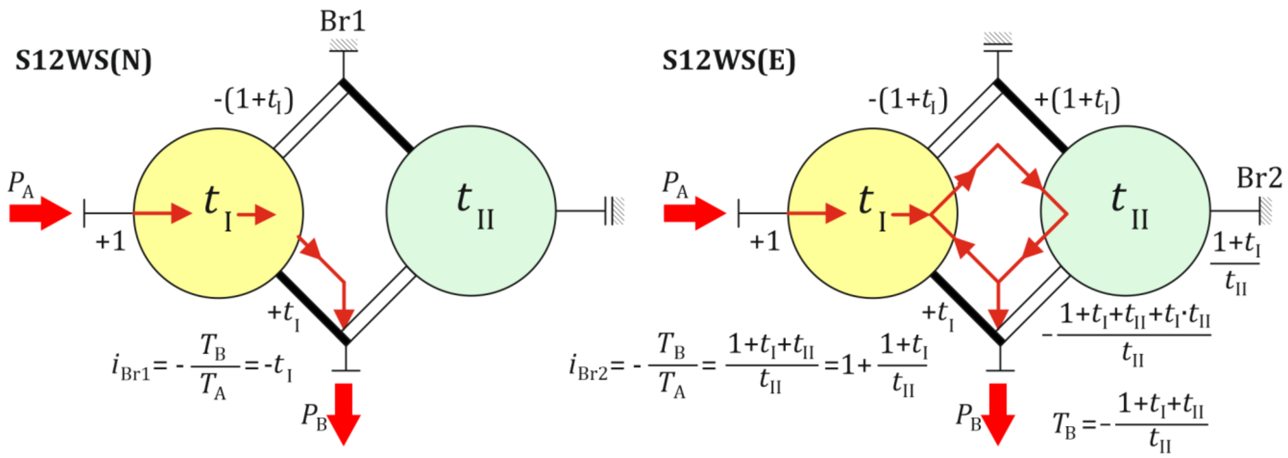

Fig. 5. Calculation of the the transmission ratio functions for the S12WS(N/E) gear train with brake Br1 (N) on and with brake Br2 (E) on

The ideal torque ratio is numerically equal to the ratio of the number of teeth of the ring gear and the sun gear for each component PGT (5,6):

$$t_I = \left| \frac{z_{3I}}{z_{1I}} \right| \tag{5}$$

$$t_{II} = \left| \frac{z_{3II}}{z_{1II}} \right| \tag{6}$$

According to the analysis of the transmission ratio functions, the PGT with brake Br1 on is a multiplicator with the output shaft running in the opposite direction to the input shaft, while with brake Br2 on the PGT becomes a reducer with the output shaft rotating in the same direction as the input shaft. It should be also noted that with brake Br1 on the PGT operates in single carrier mode with component geartrain I, while true two-carrier operation is achieved with brake Br2 on, however power circulation is present.

The kinematic scheme of complex planetary gear train S12NS(W/E) is displayed in Figure 6. This gear train retains the same basic structure of S12WS, however in this case power input and output are through external shafts N and S, while the brakes are placed on external shafts W and E.

The results of the structural analysis are displayed in Figure 7. The image on the left shows the operation of the gearset with brake Br1 (W) on as S12NS(W), while the right image shows the operation of the gearset with brake Br2 (E) on as S12NS(E).

Examination of the transmission ratio functions shows that with brake Br1 on the gear train works like a multiplier, with both the input and output shafts turning in the same direction, while with brake Br2 on the gear train becomes a reducer with both shafts also turning in the same direction. The PGT is different from S12WS(N/E) as with either brake on the PGT operates in single-carrier mode and there is no true two-carrier operation, power branching or power circulation. This kind of analysis may be applied to any layout variant discussed in this paper.

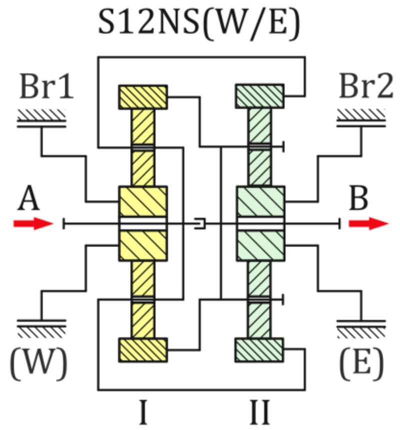

Fig. 6. Kinematic scheme of the S12NS(W/E) PGT

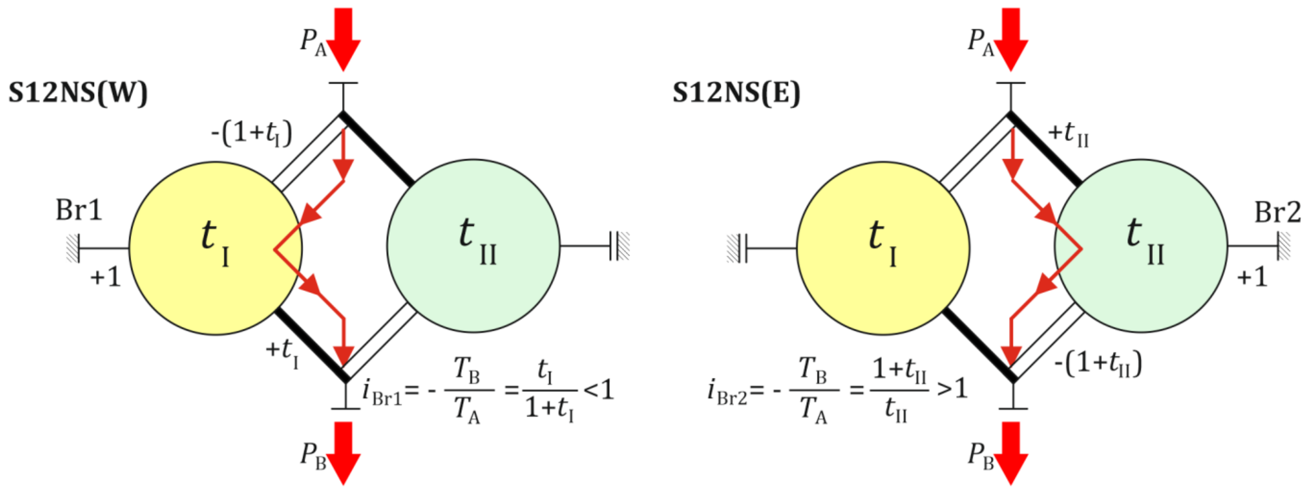

Fig. 7. Determining the transmission ratio functions for the S12NS(W/E) gear train with brake Br1 (W) on and with brake Br2 (E) on

4. Planetary Gear Trains with Multiple Variants

The number of possible transmission ratios of a planetary gear train composed of two or more component trains may be increased with a control system able to change the input and/or output shaft of the gearbox, as the changes of shaft mean that the PGT has assumed a different layout variant.

Assuming that the discussion is limited to only two-carrier PGTs, meaning two transmission ratios per layout, each compound PGT can provide up to six different transmission ratios. For example, just by enabling the prime mover and power source to be connected to two different shafts (two-variant compound PGT) it is possible to create four transmission ratios. If this is expanded to three (three-variant), it is possible to create up to six transmission ratios.

An example of a kinematic layout and structural scheme of such a PGT, S12NS(W/E)-WS(N/E) is provided in Figure 8. This PGT combines the PGTs from Figure 4 and Figure 6 in one single package. The naming convention for multivariant PGTs first names the main layout variant with the input shaft closest to the north external shaft, followed by the next external input shaft in the anticlockwise direction.

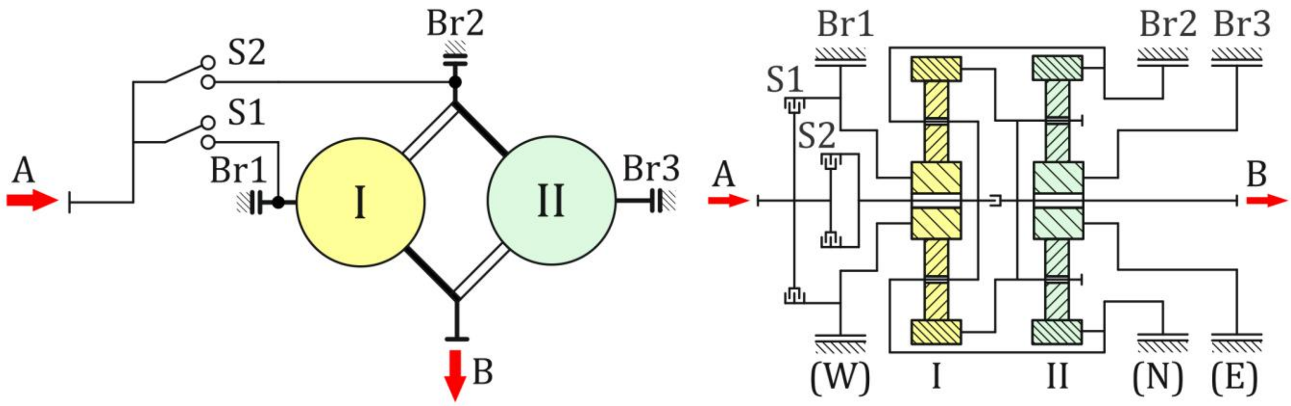

Fig. 8. Kinematic layout and structural scheme of the two-variant PGT S12NS(W/E)-WS(N/E)

This gear train is capable of four transmission ratios using a combination of clutches and brakes, and a fifth direct drive ratio with clutches only. Therefore, it is obvious that multivariant PGTs can be successfully used in practical applications, so their possibilities must be thoroughly investigated. The overview of the transmission ratios is given in Table 1.

Table 1
Overview of S12NS(W/E)-WS(N/E) transmission ratios (●/○ = control element ON/OFF)

LV	S1	S2	Br1	Br2	Br3	*i*
S12NS(W)	○	●	●	○	○	$i_{\mathrm{Br1}} = \dfrac{t_{\mathrm{I}}}{1+t_{\mathrm{I}}}$
S12NS(E)	○	●	○	○	●	$i_{\mathrm{Br3}} = 1 + \dfrac{1}{t_{\mathrm{II}}}$
S12WS(N)	●	○	○	●	○	$i_{\mathrm{Br2}} = -t_{\mathrm{I}}$
S12WS(E)	●	○	○	○	●	$i_{\mathrm{Br3}} = 1 + \dfrac{1+t_{\mathrm{I}}}{t_{\mathrm{II}}}$
DIRECT	●	●	○	○	○	$i = 1$

Kinematic analysis of the layout variants indicates that only the variants of a scheme having a common input or output shaft can be combined in a multi-variant PGT, and that shaft must remain permanently coupled to the prime mover or powered machine.

All the possible combinations of two layout variants are listed in Table 2, giving a total of 12 gear train pairs and their kinematic inversions due to input and output shafts swapping. The analysis of symbolic representations from Figure 4 based on a design standpoint has shown that only four pairs of three layout variants are viable, and those are listed in Table 3. The other combinations do not share a common input or output shaft and therefore cannot be built into a multivariant PGT.

Table 2
Theoretical combinations of two different layout variants within the same scheme for multivariant PGTs

No.	Combination	Kinematic inverse
1	WE(N/S), WS(N/E)	EW(N/S), SW(N/E)
2	WE(N/S), WN(S/E)	EW(N/S), NW(S/E)
3	EW(N/S), ES(N/W)	WE(N/S), SE(N/W)
4	EW(N/S), EN(W/S)	WE(N/S), NE(W/S)
5	WS(N/E), WN(S/E)	SW(N/E), NW(S/E)
6	SW(N/E), ES(N/W)	WS(N/E), SE(N/W)
7	SW(N/E), NS(W/E)	WS(N/E), SN(W/E)
8	NW(S/E), EN(W/S)	WN(S/E), NE(W/S)
9	NW(S/E), NS(W/E)	WN(S/E), SN(W/E)
10	ES(N/W), EN(W/S)	SE(N/W), NE(W/S)
11	SE(N/W), SN(W/E)	ES(N/W), NS(W/E)
12	NE(W/S), NS(W/E)	EN(W/S), SN(W/E)

Table 3
Feasible combinations of three different layout variants within the same scheme for multivariant PGTs (Shared input, Shared output)

No.	Combination	Kinematic inverse
1	WE(N/S), WS(N/E), WN(S/E)	EW(N/S), SW(N/E), NW(S/E)
2	EW(N/S), ES(N/E), EN(W/S)	WE(N/S), SE(S/E), NE(W/S)
3	SE(N/W), SN(W/E), SW(N/E)	ES(N/W), NS(W/E), WS(N/E)
4	NE(W/S), NW(S/E), NS(W/E)	EN(W/S), WN(S/E), SN(W/E)

5. Synthesis of Multivariant Planetary Gear Trains

The transmission ratio functions of the feasible two-speed variants are the basis for the kinematic synthesis of multivariant PGTs, as they are essentially combinations of two or three layout variants. The relations of the transmission ratios i to the ideal torque ratios t_{I} and t_{II} for each transmission ratio of every variant are provided in [22]. A graphical representation of the transmission ratio functions of a two-variant PGT will provide four stacked surfaces that share a common domain with the independent variables t_{I} and t_{II}. The graphical representation of the transmission ratios that can be obtained from a two-variant PGT that can provide four transmission ratios is provided in Figure 9 (left). The intervals of the required transmission ratios I_1, I_2, I_3 and I_4 which satisfy the condition (7) are shown on the z-axis in Figure 9 (right).

$$i_1 \in I_1, i_2 \in I_2, i_3 \in I_3, i_4 \in I_4 \tag{7}$$

A software program was developed to calculate the values of the transmission ratio functions for every possible combination of ideal torque ratios and to verify whether the value falls within the demanded range of the ratios I_1, I_2, I_3 and I_4. If such pairs of ideal torque ratios exist, the software will list them as possible solutions for further evaluation according to additional relevant criteria, i.e., diameter of component PGTs, equivalent efficiency ratio etc. [22].

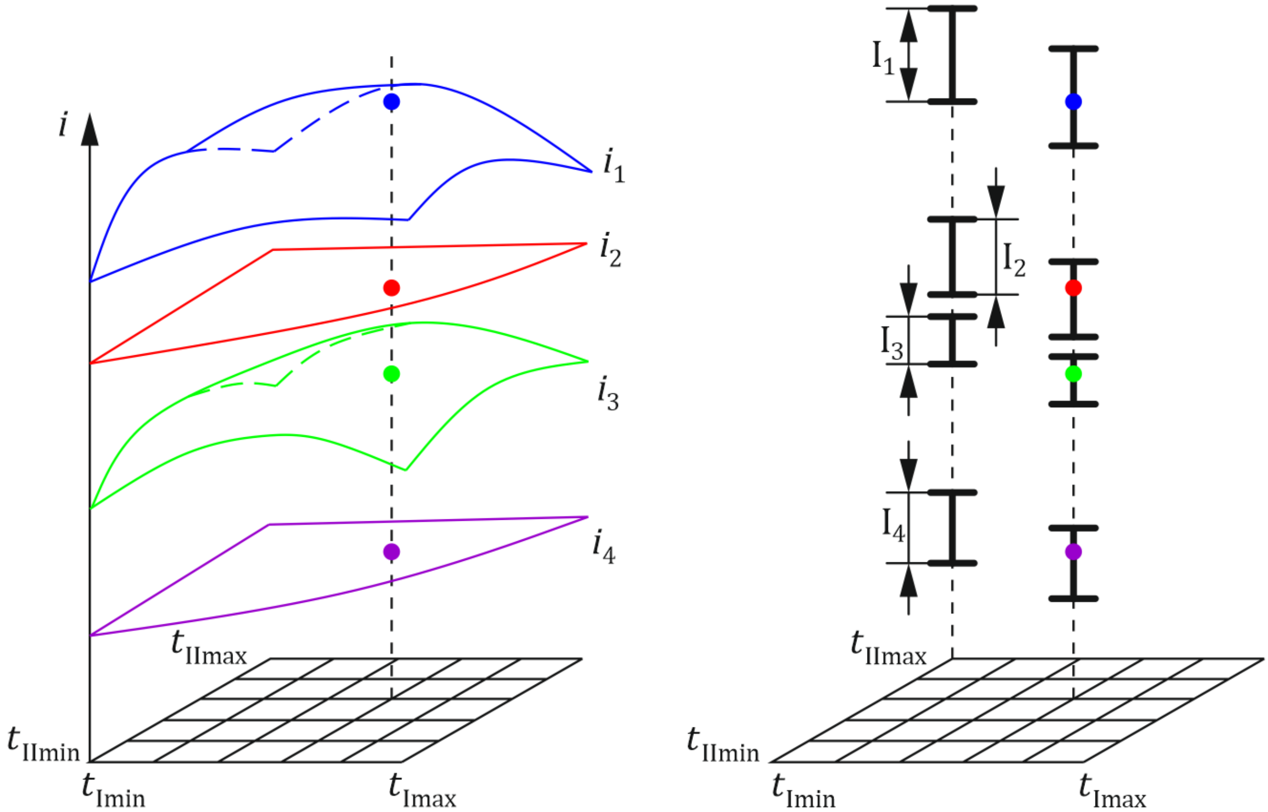

Fig. 9. Domain search procedure for detecting the combination of variants that fulfils the required transmission ratio intervals. Left: obtainable intervals. Right: overlap of the required transmission ratio intervals and layout variant capabilities

The test data from Table 4 will be used to demonstrate the operation of this program by synthetising a three-speed multivariant PGT.

Table 4
Test data for the synthesis of a three-speed multivariant PGT

Data	Value
Transmission ratio 1	$2{,}4 \leq i_{k1} \leq 2{,}6$
Transmission ratio 2	$1{,}35 \leq i_{k2} \leq 1{,}45$
Transmission ratio 3	$-2{,}7 \leq i_{k3} \leq -2{,}6$
Sun gear tooth number	$z_{1I} = z_{1II} = 18$

The software has detected six combinations (Table 5) that provide two solutions in each of the required intervals. The program has determined the exact variant and brake to be used, in addition to the data on ideal torque ratios, transmission ratios, ring gear reference diameters etc. According to the criteria of the minimal outside diameter of the component PGTs, the S36WN(S/E)-SN(W/E) gear train with $t_{\mathrm{I}} = 2{,}5$ and $t_{\mathrm{II}} = 2{,}6667$ presents an optimal solution in this case.

Table 5
Theoretical combinations of two different layout variants within the same scheme for multivariant PGTs

Scheme LV	t_I	t_{II}	i_{k1}	i_{k2}	i_{k3}	z_{3I}	z_{3II}	d_{3I} [mm]	d_{3II} [mm]
S12 EN(W/S)- SN(W/E)	2,3333	2,6667	2,571 EN(W)	1,428 SN(E)	-2,666 EN(S)	42	48	94,5	96
S12 EN(W/S)- SN(W/E)	2,5	2,6667	2,466 EN(W)	1,4 SN(E)	-2,666 EN(S)	45	48	101,25	96
S12 WS(N/E)- NS(W/E)	2,6667	2,3333	2,571 WS(E)	1,428 NS(E)	-2,666 WS(N)	48	42	96	94,5
S12 WS(N/E)- NS(W/E)	2,6667	2,5	2,466 WS(E)	1,4 NS(E)	-2,666 WS(N)	48	45	96	101,25
S36 WN(S/E)- SN(W/E)	2,3333	2,6667	2,571 WN(E)	1,428 WN(S)	-2,666 SN(E)	42	48	63	96
S36 WN(S/E)- SN(W/E)	2,5	2,6667	2,466 WN(E)	1,4 WN(S)	-2,666 SN(E)	45	48	61,87	96

The scheme and layout variant data supplied by the program was used to create the structural and kinematic scheme in Figure 10. Experience has shown that the most convenient procedure is to create the structural scheme first, and then proceed to create the kinematic scheme [6,24–26].

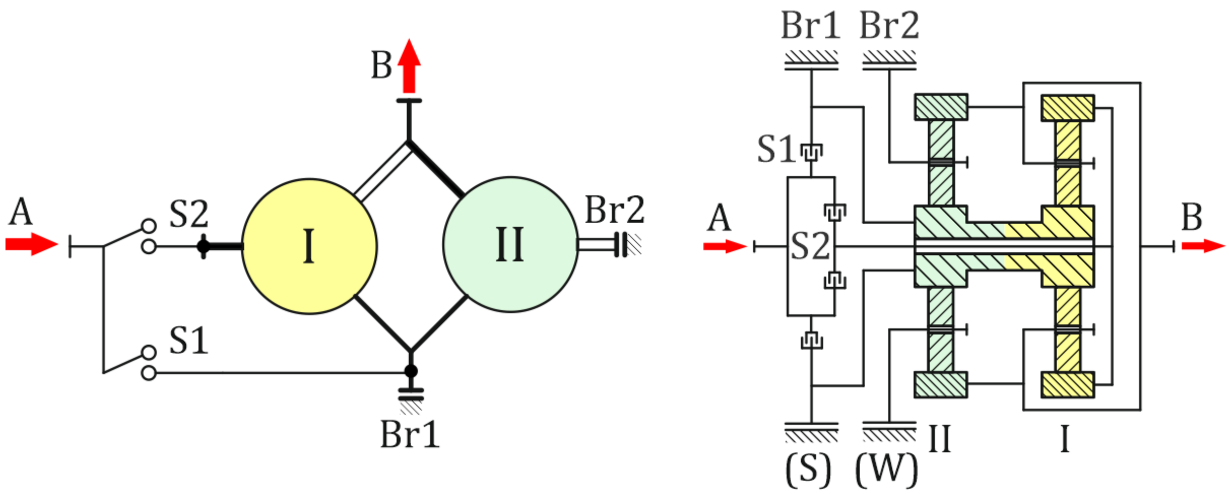

Fig. 10. Structural (left) and kinematic scheme (right) of the two-variant PGT S36WN(S/E)-SN(W/E)

6. Examples of Planetary Gear Train Element Load Function calculation

The torques acting on the basic gear train elements may be determined by conducting a separate analysis for each component PGT. To begin, a convenient sun gear is selected and assigned the torque of +1. The torques acting on the other elements are then easily calculated by means of equations (2) and (3). After the analysis of a component PGT is complete, the shaft values are transferred via the connecting shafts to the next component gearset while taking care that the torque receives an opposite sign. For example, if two planetary sets are joined by their ring gears, the torque on the second ring gear will be equal to that on the first gear, but of opposite sign.

A nodal point is defined as the location where several shafts are connected, and it is important that the sum of torques at each point must equal zero. It is also necessary to consider that the torques on the input and output members are equal to the sum of all torques related to the respective gear train member. Finally, the overall transmission ratio is then calculated using the input torque T_A and output torque T_B (8):

$$i = -\frac{T_B}{T_A} \tag{8}$$

In the case of complex, multi-carrier gear trains, the complexity of the calculation often depends on the starting point. Sometimes several starts might be needed to avoid situations with two unknown torques at a nodal point.

The torque acting on any locked element is an external torque, and it can be used as a sanity check for the calculations as the sum of torques across the shafts of any component PGT must be equal to zero.

It is possible to express all torques as a function of the input torque by multiplication with the reciprocal value of the input torque, which is then reduced to one. All further torques are then calculated by multiplying with the input torque.

6.1 The Simpson Gearset

The demonstration of the calculation procedure will be performed using a three-speed, two-variant PGT commonly called the “Simpson gearset”, which is well known in automatic transmission design since the abandonment of split-torque designs in the 1950s.

Structurally it is a S36WN(S/E)-S36SN(W/E) gear train (Figure 11), however the S36SN(W) variant may not be used as clutches S1 and S2 make it impossible to install brake Br3 required for that variant. A quick overview of the operating modes of this gearset is given in Table 6.

The transmission provides two gears in which the input and output shaft both rotate in the same direction, and one gear in which the output shaft rotates in the opposite direction. For automotive applications, a direct drive gear is obtained by activating both clutches at the same time to connect the W and S shafts together. An extra multiplication gear for fuel economy was obtained with the addition of a simple output gearset, which is easily calculated separately from the main gear train.

The gear train is commonly designed with component gearset II on the input side, and component gearset I on the output side to simplify the design of the interconnecting members. Gearset III has two valid locations, either before the clutch pack on the input side as a pre-gearbox, or more conventionally, behind gearset I. Both options have the potential for extracting six forward and two reverse gears, however the authors are unaware of such a solution being deployed at the time of writing of this article.

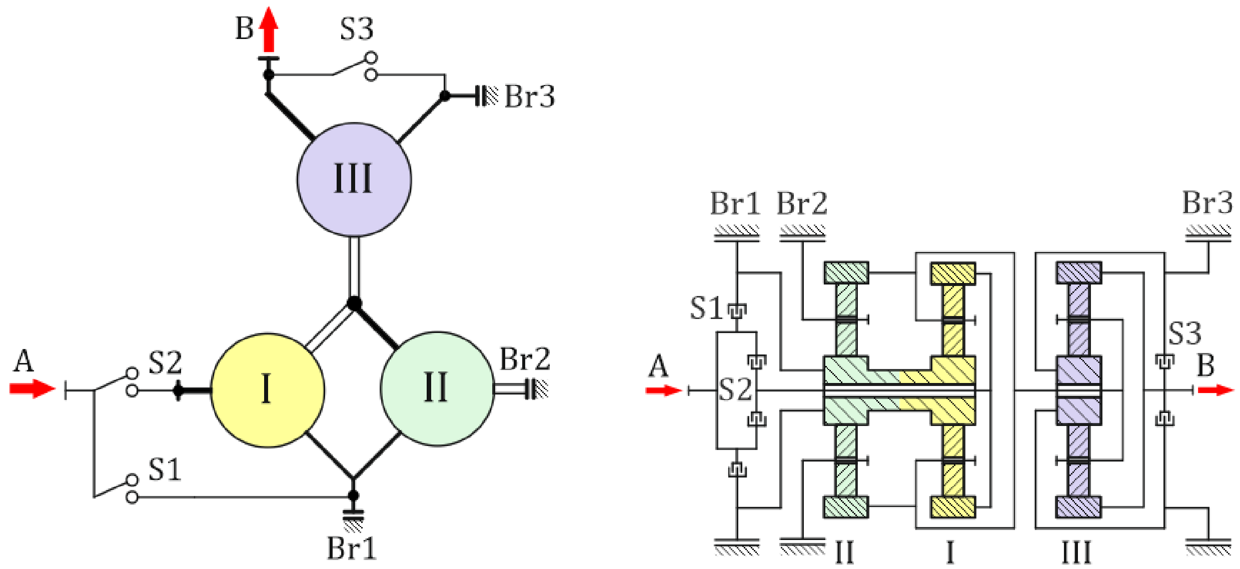

Fig. 11. Structural (left) and kinematic scheme (right) of the Simpson gearset, expanded to 4 "forward" and one "reverse" gear

Table 6
Overview of Simpson gearset based gearbox operation (●/○ = control element ON/OFF)

Gear	S1	S2	S3	Br1	Br2	Br3	*i*
1	●	○	●	○	●	○	$i_{Br2} = 1 + \frac{1+t_{II}}{t_I}$
2	○	●	●	●	○	○	$i_{Br1} = 1 + \frac{1}{t_I}$
3	●	●	●	○	○	○	$i = 1$
4	●	●	○	○	○	●	$i_{Br3} = \frac{t_{III}}{1+t_{III}}$
R	●	○	●	●	○	○	$i_{Br2} = -t_{II}$

The highest transmission ratio is achieved with layout variant S36WN(E), which is achieved by activation of clutch S1 and with brake Br2 applied (Figure 12). In this case, power input is through ring gear I after which it is split through the joined sun gear I and II shaft to PGT II before re-joining at shaft N through planet carrier I and ring gear II.

The second highest transmission ratio is achieved by shifting to layout variant S36WN(S), which is achieved by activating clutch S2 and with brake Br1 applied (Figure 13). In this case, the gear train operates in single-PGT mode through ring gear I to planet carrier I with sun gear I locked. This 1-2 change is the most difficult transition for a Simpson gearset as two brakes and two clutches must be actuated at the same time. For this reason, it is very common to pair brake Br2 with a freewheel that mimics the brake action when the engine is driving the gearbox, and to actuate the brake only if the driver selects gear 1.

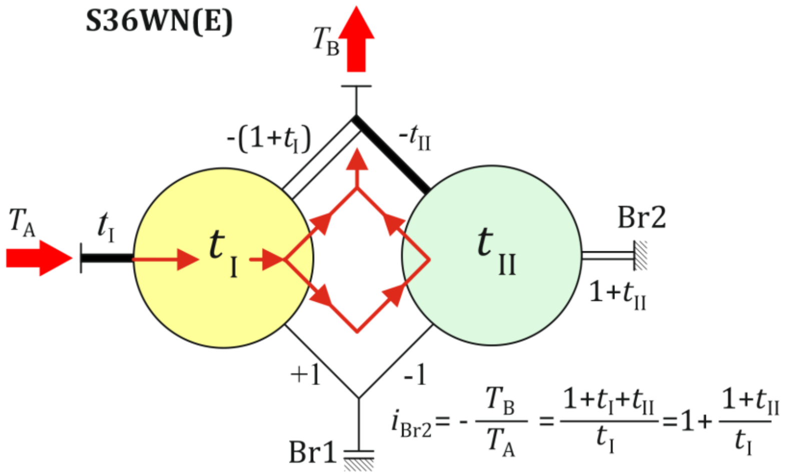

Fig. 12 Kinematic scheme of the gearset shifted into S36WN(E) mode

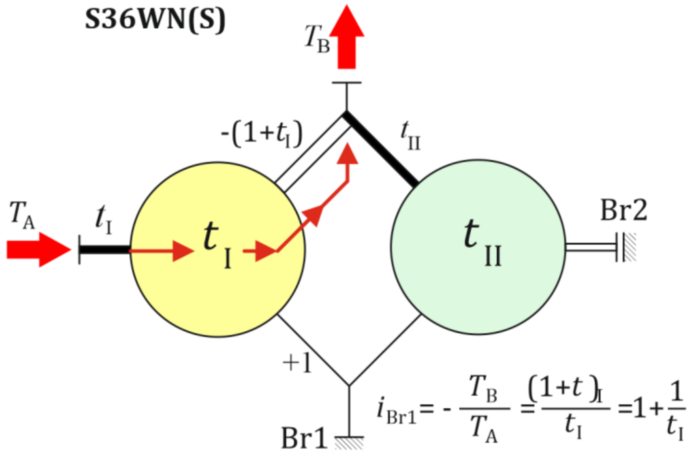

Fig. 13 Kinematic scheme of the gearset shifted into S36WN(S) mode

With layout variant S36SN(W) the output shaft rotates in the opposite direction to the input shaft, which is very convenient for an automotive reverse gear (Figure 14).

This variant is achieved by engaging clutch S1 with brake Br2 on. Power is transmitted in single-PGT mode from sun gear II to ring gear II with carrier II held stationary, causing the output element to rotate in the direction opposite to the input shaft.

The gearset is capable of another transmission ratio as layout S36SN(E) (Figure 15). In this mode, the power enters the gearset through external shaft S, enters PGT I via the sun gear, and exits PGT I to external shaft N via planet carrier I. However, this transmission ratio is not kinematically practicable as it renders impossible the installation of the clutch pack with clutches S1 and S2 (Figure 12). The gear train also has a direct drive mode that is achieved by activating simultaneously both the clutches S2 and S1 while both brakes are released, locking the ring and sun gears of PGT I to cause the whole gear train to rotate in unison.

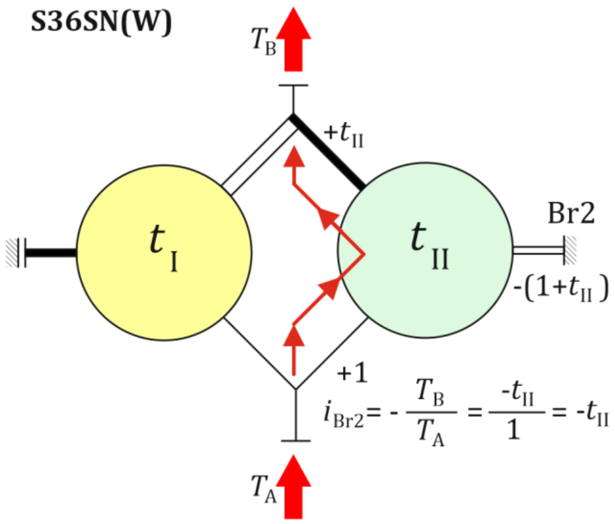

Fig. 14 Kinematic scheme of the gearset shifted into S36SN(W) mode

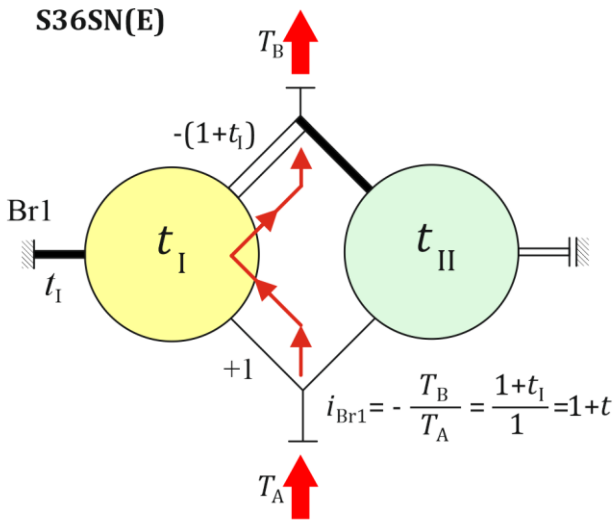

Fig. 15 Kinematic scheme of the gearset shifted into S36SN(E) mode

The third gear train (PGT III) is present only in newer iterations of the gearbox. It was added to promote fuel economy with a transmission ratio in which the output shaft rotates faster than the input shaft.

The gear train is normally held locked by clutch S3, connecting the sun and ring gears, however when fourth gear is engaged, this clutch is released together with brake Br3 locking the sun gear III, increasing the speed of the output shaft.

Even though the original design is somewhat dated, the transmission is extremely robust and proven to a point that a variant of the Simpson gearset with the 4th gear PGT added was used by Porsche as a the ZF4HP22HL automatic with manual control (Tiptronic) gearbox, with a special rapid-action valve body design [27,28].

6.2 The Wilson Epicyclic Gearbox

The Wilson epicyclic gearbox contains one of the oldest known working planetary gearset designs. It uses four component PGTs, four brakes and one clutch to provide three forward gears, one direct drive gear, and one reverse gear. It presents an extremely sturdy design that uses only brakes to

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change gears, except for direct drive where a clutch is used. As the gearbox was specifically designed as a tank transmission before being adapted to road use, it is also extremely easy to operate. Nowadays, it can still be found, combined with a fluid coupling or a torque converter, as a bus or light railcar transmission. Even though the gearbox was originally a semiautomatic design, it is possible to build a fully automated Wilson box. The only downside of this design is that it obtains only three forward gears (excluding direct drive) and one reverse gear from a gearset containing four component PGTs (Figure 16).

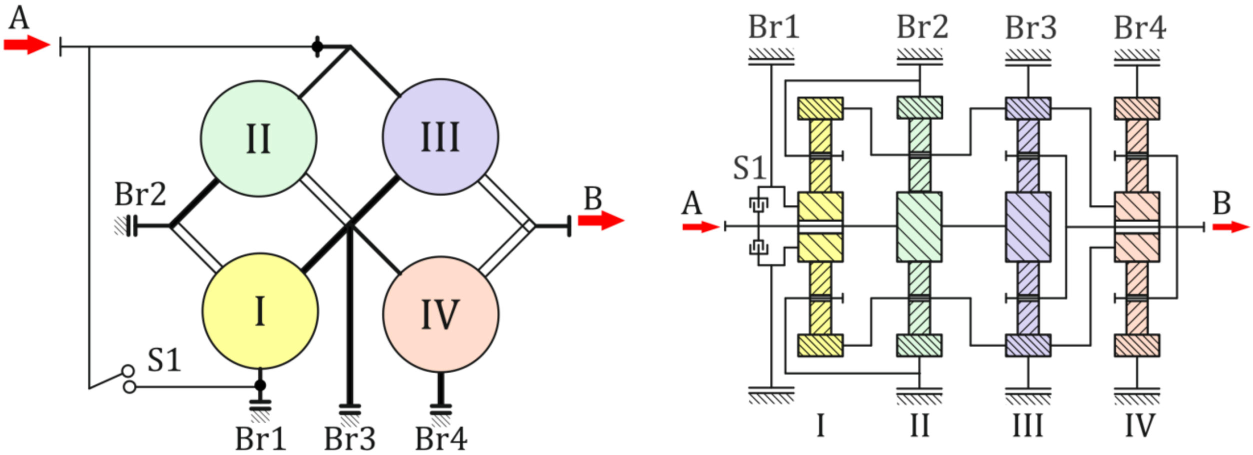

Fig. 16 Structural (left) and kinematic scheme (right) of the Wilson gearset

The operating principle of the gearset is provided in Table 7. Two control elements need to be actuated to perform a gear change, enabling very smooth operation.

Table 7
Overview of Wilson epicyclic gearbox operation (●/○ = control element ON/OFF)

Gear	S1	Br1	Br2	Br3	Br4	*i*
1	○	○	○	●	○	$i_1 = 1 + t_{\mathrm{III}}$
2	○	○	●	○	○	$i_2 = \dfrac{1+t_{\mathrm{II}} \quad 1+t_{\mathrm{III}}}{1+t_{\mathrm{II}}+t_{\mathrm{III}}}$
3	○	●	○	○	○	$i_3 = \dfrac{t_{\mathrm{II}}+t_{\mathrm{III}} \quad 1+t_{\mathrm{I}}+t_{\mathrm{II}}}{1+t_{\mathrm{I}} \quad 1+t_{\mathrm{III}} \quad +t_{\mathrm{II}}}$
4	●	○	○	○	○	$i_4 = 1$
R	○	○	○	○	●	$i_{\mathrm{R}} = 1 - t_{\mathrm{III}} t_{\mathrm{IV}}$

For first gear, brake Br3 is activated, sending gear train III into single carrier operation. Input is sun gear III, output to carrier III.

Second gear is obtained by activating brake Br2. This locks ring gear II, causing trains II and III to operate as S36SE(W). Input is through sun gears II and III, output to carrier III.

Third gear is obtained by activation of brake Br1. This locks sun gear I, sending gear trains I, II and III into three-carrier operation. Power input is through sun gears II and III, output to carrier III.

In fourth gear, clutch S1, is activated to connect sun gear I to sun gears II and III, causing the gear train to rotate as a block, providing direct drive.

Reverse gear is obtained by activating brake Br4, sending the gear train into two-carrier mode as S13WN(E). Input is to sun III, output from combined carriers III and IV.

Even though this gearset design is dated, it is a clear example of a gearbox designed with reliability in mind, and the analysis of the structure of the Wilson gearset reveals similarities with the gearset used in the ZF HP 500 gearbox family, notably the permanent interconnections and brake layout. This is especially easy to observe in the 4HP500 variant, so that the HP500 family could be considered as an evolved version of the Wilson box replacing one brake with a clutch, with the reverse gear derived through the interconnections of the component gear trains.

6.3 The ZF HP500 Gearbox Family

The ZF HP500 is a family of gearboxes built for mid to high power applications in city and highway buses. It is clearly designed for high loads, as most of the shifting is done by three or four brakes depending on the model, and two or three clutches. The base model obtains six forward gears and one reverse gear from a three-PGT gearset (Figure 17).

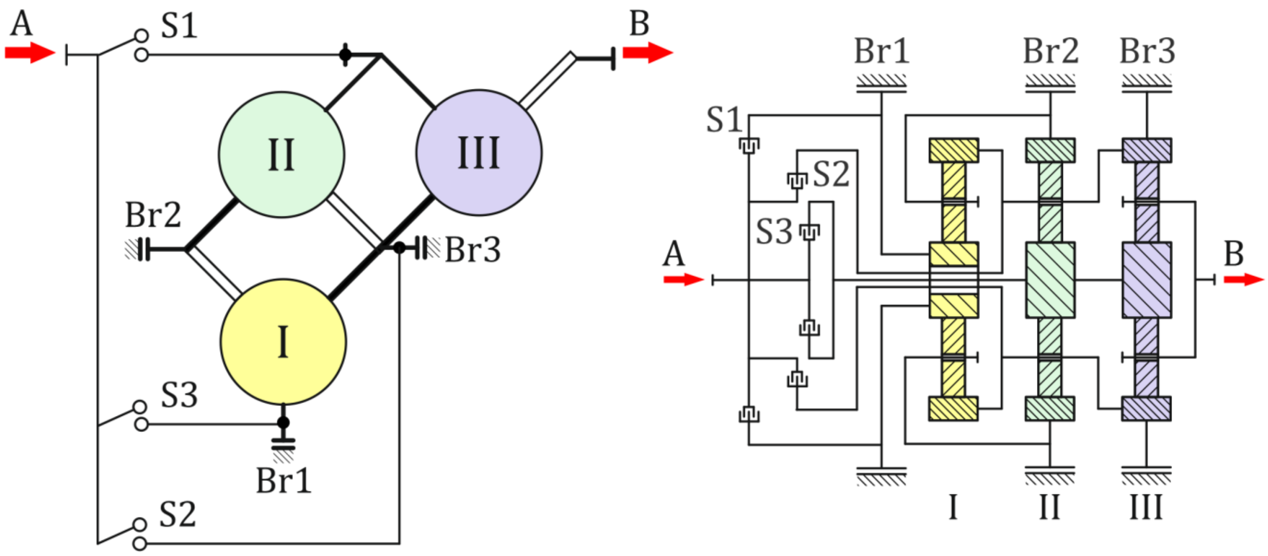

Fig. 17 Structural (left) and kinematic scheme (right) of the three-carrier, six-speed ZF 6HP500 gearbox

Most interconnections are made through planet carriers and ring gears, with sun gears II and III sharing a common shaft. The same gearbox exists with four (4HP500) and five (5HP500) gears. In the five-gear model, sixth gear is disabled in the control system, the four-gear version deletes clutch S2 connecting the input shaft to rings I and III and carrier 2 (Figure 18). A quick overview of the operating modes of this gearbox is given in Table 7.

In first gear, clutch S3 and brake Br3 engage the gearbox in single carrier mode over PGT III. For second gear, clutch S3 and brake Br2 engage PGTs II and III into two-carrier mode as S36SE(W). Third gear is achieved by engaging clutch S3 together with brake Br1, resulting in three-carrier operation of PGTs I, II and III.

In fourth gear, clutches S3 and S2 (or S1 and S3 for the 4HP500) lock PGT III in unison, providing direct drive with $i_4 = 1$.

In fifth gear, clutch S2 drives the interconnected carrier and ring gears while brake Br1 engages to lock sun gear I, operating the gear train in three-carrier mode.

For sixth gear, brake Br2 and clutch S2 are activated at the same time, sending PGTs II and III into two-carrier mode as S36NE(W).

Table 7
Overview of ZF 4/5/6HP500 three-carrier gearbox operation (●/○ = control element ON/OFF)

Gearbox	Gear	S1	S2	S3	Br1	Br2	Br3	*i*
ALL	**1**	○	○	●	○	○	●	$i_1 = 1 + t_{III}$
ALL	**2**	○	○	●	○	●	○	$i_2 = \frac{1 + t_{II} + t_{II}t_{III} + t_{III}}{1 + t_{II} + t_{III}}$
ALL	**3**	○	○	●	●	○	○	$i_3 = \frac{1 + t_I + t_{II} + t_{III} + t_I t_{III} + t_{II}t_{III}}{1 + t_I + t_I t_{II} + t_{III}}$
4HP500	**4**	●	○	●	○	○	○	$i_4 = 1$
5/6HP500	**4**	○	●	●	○	○	○	$i_4 = 1$
5/6HP500	**5**	○	●	○	●	○	○	$i_5 = \frac{t_{II} + t_{III} + t_I t_{III}}{t_I t_{II} - t_{II} - 1 - t_I + t_{III} + t_I t_{III}}$
6HP500	**6**	○	●	○	○	●	○	$i_6 = \frac{1 + t_{III}}{1 + t_{II} + t_{III}}$
ALL	**R**	●	○	○	○	○	●	$i_R = -\frac{1 + t_I + t_{III} + t_I t_{III}}{t_{II}}$

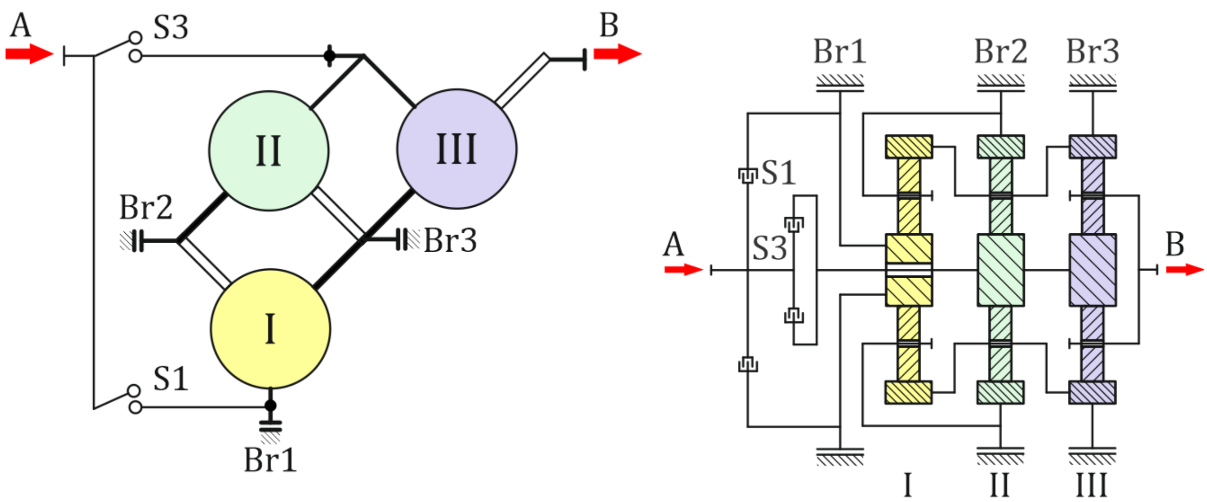

Fig. 18 Structural (left) and kinematic scheme (right) of the three-carrier, four-speed ZF 4HP500 gearbox, note that the clutches are deliberately marked as S1 and S3 for continuity with Figure 17, Figure 19 and Table 7

Reverse gear is obtained by activating brake Br3 and clutch S1, operating PGTs I, II and III as three serially connected simple PGTs. Carrier I outputs to ring II, carrier II is locked, which causes sun II to turn sun III opposite to sun I.

The gearbox is also manufactured in a heavy-duty variant with four gearsets and six forward gears, and the operating modes of this gearbox are listed in Table 8.

Table 8
Overview of ZF 6HP500 four-carrier gearbox operation (●/○ = control element ON/OFF)

Gear	S1	S2	S3	Br1	Br2	Br3	Br4	*i*
1	○	○	●	○	○	○	●	$i_1 = 1 + t_{\mathrm{IV}}$
2	○	○	●	○	○	●	○	$i_2 = 1 + t_{\mathrm{III}}$
3	○	○	●	○	●	○	○	$i_3 = \dfrac{1 + t_{\mathrm{II}} + t_{\mathrm{II}} t_{\mathrm{III}} + t_{\mathrm{III}}}{1 + t_{\mathrm{II}} + t_{\mathrm{III}}}$
4	○	○	●	●	○	○	○	$i_4 = \dfrac{1 + t_{\mathrm{I}} + t_{\mathrm{II}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}} + t_{\mathrm{II}} t_{\mathrm{III}}}{1 + t_{\mathrm{I}} + t_{\mathrm{I}} t_{\mathrm{II}} + t_{\mathrm{III}}}$
5	○	●	●	○	○	○	○	$i_5 = 1$
6	○	●	○	●	○	○	○	$i_6 = \dfrac{t_{\mathrm{II}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}}}{t_{\mathrm{I}} t_{\mathrm{II}} - t_{\mathrm{II}} - 1 - t_{\mathrm{I}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}}}$
R	●	○	○	○	○	●	○	$i_{\mathrm{R}} = -\dfrac{1 + t_{\mathrm{I}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}}}{t_{\mathrm{II}}}$

This variant has an extra gearset IV which is literally bolted onto gearset III, and an extra brake Br4 is added. This gearset IV is used as first gear, and the gears from 1 to 5 of a standard gearbox become gears 2 to 6 in the four gearset model (Figure 19) The first gear calculation in this case is numerically equal to calculation for the three-gearset variant.

It is interesting that this gearset is capable of seven gears, but the seventh gear remains disabled in the control unit. As with the three-carrier model, consecutive shifts are made by actuating only two control elements, making for very smooth operation, which is expected from a bus gearbox.

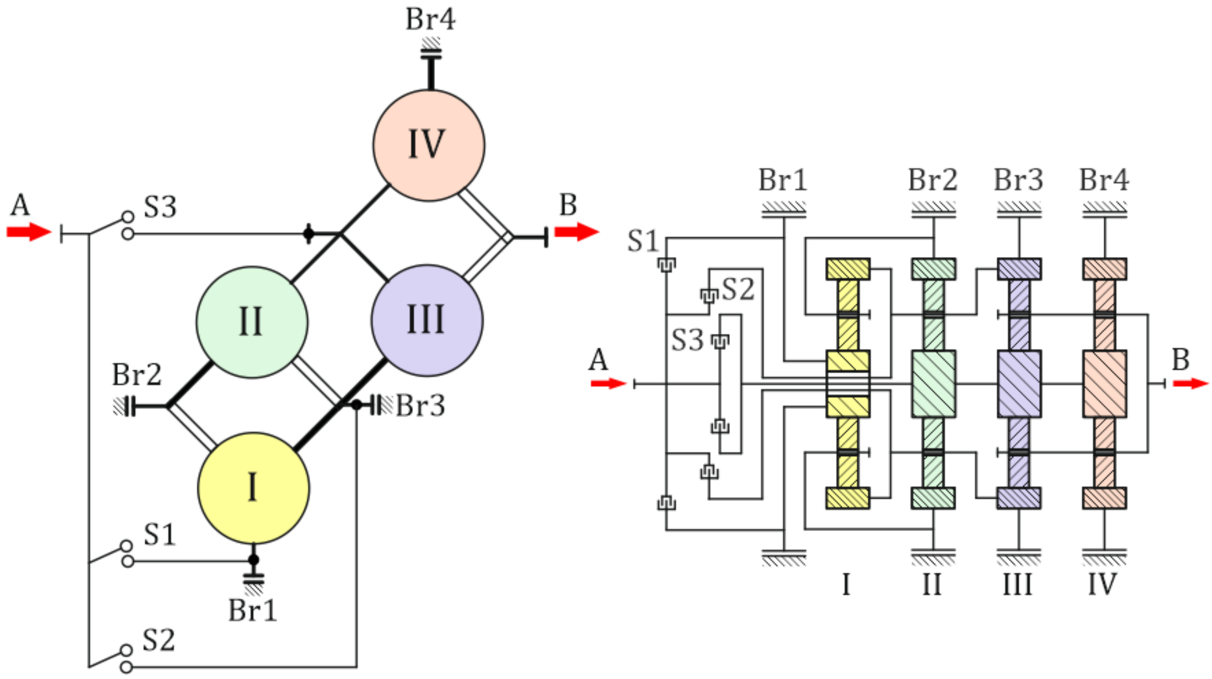

Fig. 19 Structural (left) and kinematic scheme (right) of the four-carrier, six-speed ZF 6HP500 gearbox

6.4 The ZF 8HP Gearbox Family

The ZF 8HP is a contemporary gearbox family for mid to high powered road applications, with uprated torque input for mild hybrid applications using a "pancake" motor generator bolted to the engine flywheel. It is a good example (Figure 20) of a recent trend in planetary gearbox design, characterized by the abandonment of Lepelettier and Ravigneaux sets due to the high torque inputs provided by electrically assisted internal combustion engines.

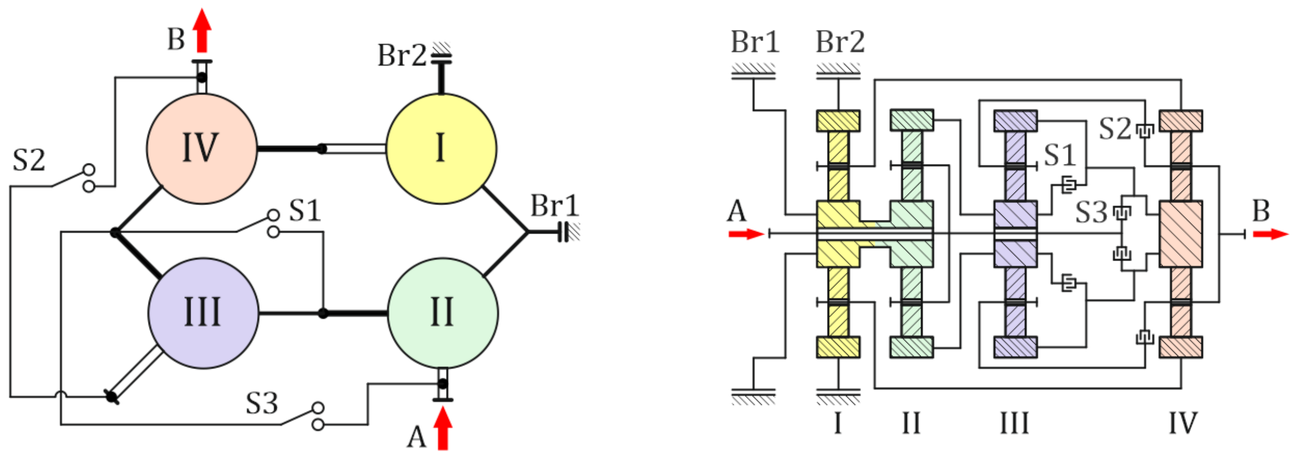

Fig. 20 Structural (left) and kinematic scheme (right) of the four-carrier, eight-speed ZF 8HP70 gearbox

The 8HP family of gearboxes is characterized by four component gearsets, of which I and II have interconnected suns, ring II permanently connected to sun II, sun III permanently connected to sun IV, and carrier I permanently connected to ring gear IV. There are two brakes in the design, Br1 acting on connected suns I and II, and brake Br2 acting on ring I. Carrier II is permanently connected to the input shaft, while clutch S3 connects the input shaft to the link between ring III and sun IV. This link is also connected to sun III and ring II through clutch S1. Finally, clutch S2 connects planet carrier III to planet carrier IV. As it is a car transmission, most of the shifting is done by clutches. The operating modes of the ZF 8HP70 gearbox are listed in Table 9.

First gear is achieved by engaging brakes Br1 and Br2, locking ring IV, while clutch S3 connects sun gear IV to the input shaft, effectively engaging the gearbox in single carrier mode over PGT IV.

In second gear, brakes Br1 and Br2 engage to lock sun II and ring gear IV. Input is via carrier II to ring II, and clutch S1 completes the connection to the sun gear IV. The gearbox effectively operates as a combination of two linearly joined PGTs.

In third gear, brake Br2 is engaged stopping ring gear I. Clutch S3 connects sun gear IV to the input shaft, while clutch S1 connects ring gear II to the input shaft, causing PGT 2 to turn like a block. This in turn causes PGTs I and IV to operate in two-carrier mode as S36SE(W).

In fourth gear, brake Br2 holds ring gear I, while the application of clutches S1 and S2 causes gearsets III and IV to rotate like a block, while also connecting carrier I to ring gear II. Input is through carrier II, and there is a constant connection between suns I and II. This in turn causes PGTs I and II to operate in two-carrier mode as S36EN(W).

In fifth gear, brake Br2 locks ring gear I. Clutch S3 connects sun gear IV and ring gear III to the input shaft, and clutch S2 connects carrier III to carrier IV. Carrier II then drives the sun gears II and I which in turn drive carrier I, operating the gear train in true four-carrier mode.

In sixth gear, clutches S1, S2 and S3 engage to connect PGTs III and IV as a block to the input shaft. This results in the PGT being locked in direct drive with.

Table 9
Overview of ZF 8HP70 four-carrier gearbox operation (●/○ = control element ON/OFF)

Gear	S1	S2	S3	Br1	Br2	*i*
1	○	○	●	●	●	$i_1 = 1 + t_{IV}$
2	●	○	○	●	●	$i_2 = \frac{t_{II} \;\; 1 + t_{IV}}{1 + t_{II}}$
3	●	○	●	○	●	$i_3 = \frac{1 + t_I + t_I t_{IV} + t_{IV}}{1 + t_I + t_{IV}}$
4	●	●	○	○	●	$i_4 = \frac{1 + t_I + t_{II}}{1 + t_{II}}$
5	○	●	●	○	●	$i_5 = \frac{1 + t_{IV} + t_{II} t_{IV} \;\; 1 + t_{III}}{1 + t_I + t_{IV} \;\; 1 + t_{III} \;\; 1 + t_{II}}$
6	●	●	●	○	○	$i_6 = 1$
7	○	○	●	●	○	$i_7 = \frac{t_{II} \;\; 1 + t_{III}}{t_{II} \;\; 1 + t_{III} \;\; + 1}$
8	●	●	○	●	○	$i_8 = \frac{t_{II}}{1 + t_{II}}$
R	○	●	○	●	●	$i_R = \frac{-t_I t_{III} \;\; 1 + t_{IV} \;\; + t_{II} \;\; 1 + t_{III}}{1 + t_{II}}$

For seventh gear, brake Br1 locks sun gears I and II, while clutch S3 connects ring gear III to the input shaft. Input is through carrier II which drives ring gear II and in turn sun gear III. Output is through gearset IV which rotates as a block with carrier III. This results in gearsets II and III operating as a two-carrier train S16NE(W).

Eighth gear is achieved with Br1 holding sun gears I and II, with power flow from carrier II to ring II. Clutches S1 and S2 engage to cause PGTs III and IV to rotate in unison with ring gear II.

Reverse is obtained by activating brakes Br1 and Br2 to hold sun gears I and II while also locking carrier I, and by consequence, ring IV. Clutch S2 engages to connect ring gear III to the output shaft, causing a reversal of rotation. Ring gear III is connected to sun IV, driving PGT IV in the opposite direction of the input shaft. Analysis shows that in this case, the gear train is configured as PGT II operating as a carrier to ring multiplier connected to trains III and IV operating as two-carrier train S13WN(E).

7. Conclusion

This paper deals with the procedures for the calculation of two-carrier, two-variant switching planetary gear trains. These gear trains enable the creation of gearboxes with four or less transmission ratios plus direct drive by means of a two-carrier gear train. This procedure was extended to three-variant PGTs by extended analysis of the properties of two and three variant trains, thus providing a maximum of six transmission ratios plus direct drive.

A lot of PGTs satisfy this general description, so the design requires a methodical approach or software support to avoid suboptimal solutions. The kinematic characteristics of multivariant planetary gear trains are thoroughly analysed, and a procedure for their classification is proposed in combination with the methods for planetary gear train synthesis.

This procedure covers the operating regimes of every gear drive, enabling the designer to select only the variants that fulfil the application demands. The torque method is an important tool in these calculations, and it has been presented in this paper and thoroughly explained on the example of a Simpson gearset.

Three additional gearsets have been analysed to better illustrate the procedure. The first is the so-called Wilson gearset, which can be considered the progenitor of all heavy-duty planetary gear trains, the second a heavy-duty commercial gearbox, and the final a somewhat lightly built box for heavy torque loads as exhibited by hybrid motor vehicles duty. All PGTs have been analysed to demonstrate the differences in the design approach and reduced to their actual operating units for every transmission ratio.

It was effectively proven that most gearsets operate in two-carrier mode in most cases, and that simultaneous operation with more than three PGTs is generally avoided by designers, most probably due to the complexity of the calculation, or due to power circulation or transmission elements being subject to extreme centrifugal loadings. Four-carrier mode is usually restricted to few occasions for gearsets that must extract many transmission ratios from a small number of gearsets. Single-carrier mode is usually reserved for low gears in heavy duty boxes, or for overdrive gears in both heavy duty and general applications

It can be concluded that the calculation methods presented in this paper are applicable to various gear trains, including those with two and more than two planet carriers, and that the methods for the selection of optimal gear train configurations presented in this paper may be successfully applied in the future development of computer programs.

Author contributions

Conceptualization, S.T.; methodology, S.T.; software, S.T.; validation, K.M., Ž.V. and D.M.; formal analysis, D.M. and K.M.; investigation, Ž.V; resources, Ž.V.; data curation, D.M.; writing—original draft preparation, Ž.V.; writing—review and editing, S.T. and K.M.; visualization, Ž.V.; supervision, S.T. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

The original research data is available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The funder had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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