Spectrum of Mechanical Engineering and Operational Research, Volume 2, Issue 1 (2025) 24-46

SCIENTIFIC OASIS

Spectrum of Mechanical Engineering
and Operational Research

Journal homepage: www.smeor-journal.org
eISSN: 3042-0288

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Spectrum of Mechanical Engineering and Operational Research

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Investigation of the Settlement Behavior of Ballasted Railway Tracks Due to Dynamic Loading

Szabolcs Fischer[1,*]

[1] Department of Transport Infrastructure and Water Resources Engineering, Széchenyi István University, Győr, Hungary

ARTICLE INFO

Article history:
Received 4 July 2024
Received in revised form 3 December 2024
Accepted 20 December 2024
Available online 22 December 2024

Keywords:
Railway ballast; Settlement models; Dynamic loading; Track geometry; Ballast compaction.

ABSTRACT

This study investigates the settlement behavior of ballasted railway tracks under dynamic loading, providing a comprehensive evaluation of track deterioration models using extensive field data. Leveraging regression analysis, the research examines logarithmic and exponential settlement trends based on operational conditions and highlights key influences, such as ballast compactness and maintenance interventions. Validated against historical ORE (Research and Testing Office) studies, the results suggest refinements in constants to better predict contemporary track behavior. The integration of advanced computational and experimental techniques is proposed to improve model accuracy, ensuring effective maintenance planning and enhanced track durability.

1. Introduction

Since the first industrial revolution, rail transport has played a critical and prominent role in the history of the world and is still one of the most important modes of transport and transportation between the countryside and the capital, as well as between the big cities and the capital(s). Thanks to its low resistance and high traction, it can haul large masses of freight, transport heavy goods trains at moderate speeds or passengers at speeds of up to several hundred km/h or serve local public transport needs on the "surface" (e.g., tramway, light rail, elevated railway, etc.) or underground (e.g. metro, subway, etc.). In addition, there are specialized "modes" of rail transport, such as funiculars, rack railways, cableways, etc. [1].

It is worth noting that the operation of railways requires not just one sector but all sectors. These include other disciplines such as civil engineering [2-5], mechanical engineering [2,3,6-8], electrical engineering [9,10], transportation engineering [11-14], information technology [13], etc. The disciplines are the railway track and structures (bridges, tunnels, retaining walls, etc.), rolling stock and traction, traffic management, logistics, telecommunications, power, safety and signaling, etc.

More than 95% of the world's railways are of traditional crushed stone ballasted construction, while the remaining 5% or so are of ballastless construction [15]. The superstructure (which, if of

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[*] *Corresponding author.*
E-mail address: fischersz@sze.hu

https://doi.org/10.31181/smeor21202528

ballasted design, consists of the track – i.e., the rails, rail fastenings, sleepers – and the crushed stone ballast) is supported by the railway substructure, which is usually made of compacted soil material.

One of the advantages of ballasted railway tracks is that if settlement (i.e., plastic vertical deformations) in the tracks is not due to substructure failure, it is possible to restore the track level to the condition at the time of construction in a strictly theoretical infinite number of times, up to a position where the track geometry can be maintained within the required dimensional limits when measured using proven standard track diagnostic methods [1]. Geometrical correction work can be executed by manual force or by hand-machine or large-machine methods. In contrast, the disadvantage of ballasted railway tracks is precisely the contradiction with the theoretical assumption, i.e., the real, practical result. Several factors play a role in this, e.g., (i) even the vehicle load, (ii) even the ballast compaction and tamping, (iii) even the environmental (weathering) effects, which cause the crushed stone ballast grains to wear, fragment, fracture (in extreme cases, crumbling) [16]. These result in a change in the original particle size distribution (PSD) of the aggregate (more fine grains and smaller diameter grains), breakage of sharp edges, and a reduction in the internal angle of friction of the aggregate, and hence its internal/inner shear resistance [17]. Due to these reduced characteristics, the plastic deformation of the aggregate (sample) is increased in response to both static and dynamic forces. Of course, perfect, dust-free, cubic-sharp cornered-sharp edges of grains will also deform under such forces, but typically to a significantly smaller extent. One of the most critical characteristics of the system is the compactness (or density) of the granular aggregate (sample). This compactness is relatively easy to measure and determine in a granular material with a continuous grain distribution (e.g., sandy gravel, soil fill, etc.). There are standard methods for this, e.g., isotopic density measurement [1]. In the case of railway ballast, the measurement of density is extremely difficult, typically impossible. This is due to the predominance of large grains (31.5-40-50-50-63-80 mm) and the minimum number of small (<31.5 mm) and fine (<0.5 mm) grains (<0.063 mm) [18]. On the other hand, the compactness of the aggregate (sample) is very important for the support and mechanical behavior of the track. Compaction can be achieved either with a ballast (bed) compactor (inter-sleeper compactor, ballast shoulder compactor) or with hand-held or large machine tamping tines/tamping machines, and a dynamic track stabilizer is a typical technical solution - but this is mainly used as a complementary solution. It is not possible to control the compaction during the working process, and it is not possible to carry out precise control measurements after the working process [1].

In addition to the above, the impact of the substructure is also significant. By this, it is meant that a substructure with a low load-bearing capacity of the soil material (mainly without or sometimes with a granular sub-base/supplementary layer), or a poor-quality embankment foundation, or inadequate drainage (or any combination of these) can have the effect that the geometric stability of the track is not or cannot be solved [1]. The tangible result of these is that the track level is ''unsteady'', with rapid and continuous geometrical deterioration despite successive track geometry corrections and tamping, i.e., this relatively rapid deterioration cannot be stopped. In some of these cases, the phenomenon observed is only localized, i.e., it is limited to shorter lengths (a few meters or a few tens of meters). In such cases, the first suspected cause is a poorly load-bearing sub-structure, possibly the absence of an additional layer, or an unresolved local drainage problem [1]. The lack of a supplementary layer can cause the crushed stone bedding grains to be pressed into the substructure and mixed with the upper layer, and lateral drainage and water evacuation at these local points becomes impossible and problematic, resulting in the formation of so-called water pockets or mudflats. These problems can be solved, usually by ballast screening, partial soil replacement, installation of a granular supplementary layer (with or without geosynthetics), etc. [1].

The plastic vertical deformation of ballasted railway tracks changes under the action of the load (i.e., the rolled-over railway axles or, more simply, the through-rolled axle tons in the MGT unit, which means "million gross tons"). A cumulative settlement can be detected, usually described by a logarithmic function of the trend [1]. This logarithmic regression function is most characteristic in the case where the substructure's load-bearing capacity is adequate and drainage is not a problem. In the case where there are support or load-bearing capacity problems, this trend may even become an exponential trend. International literature provides a considerable number of different and divergent trend functions for determining settlement. There are also significant differences between them [1].

This paper aims to review the trends in international literature, mainly logarithmic, using a given validated data set of several years. The structure of the paper is as follows: a detailed international literature review is presented in Section 2, Section 3 discusses materials and methods, Section 4 presents the results and discussion, and Section 5 summarizes the conclusions.

2. Literature review

The settlement of ballasted railway tracks is a key challenge in railway engineering because it directly affects track geometry, safety, and overall efficiency. Over the years, researchers have developed various models to predict and understand settlement under different loading conditions, each offering unique strengths and practical applications. In the below subsections, the author of the current paper aimed to compose a compact review that brings together insights from studies that have applied different mathematical approaches, such as logarithmic models, power functions, and others, to address the complexities of track settlement.

2.1 Models based on logarithmic functions

The foundational work by Xu *et al.* [19] established a laboratory test facility that examined the relationship between the resilient modulus of the subgrade and the settlement of ballasted tracks. Their findings highlighted the importance of understanding the mechanical behavior of ballast under cyclic loading, which is crucial for developing accurate predictive models. Similarly, Zhou *et al.* [20] conducted a full-scale experimental study that demonstrated a logarithmic relationship between the number of load cycles and the rate of settlement in ballasted tracks, indicating that as the number of cycles increases, the rate of settlement tends to stabilize, following a logarithmic trend.

In the context of dynamic loading conditions, Kaewunruen and Tang [21] explored the dynamic behavior of railway ballast under flooding conditions, providing insights into how moisture affects settlement characteristics. Their findings suggest that the dynamic response of ballast can be modeled using logarithmic functions to account for the varying degrees of saturation and its impact on settlement. This is particularly relevant in regions prone to flooding, where the ballast's mechanical properties can significantly change, leading to increased settlement.

Further research by Li *et al.* [22] utilized a three-dimensional modeling approach to investigate differential railway track settlement. They employed a cycle domain constitutive model that incorporated logarithmic functions to describe the relationship between stress and settlement over time. This model effectively captured the nonlinear behavior of ballast under repeated loading, providing a robust framework for predicting long-term settlement trends.

Varandas *et al.* [23] focused on the settlement of ballasted tracks under traffic loading, particularly in transition zones. Their methodology involved dynamic calculations that included logarithmic models to account for the incremental nature of settlement as a function of loading cycles. The study emphasized the significance of understanding the non-linear aspects of ballast behavior, which can be effectively captured through logarithmic functions.

Park and Lim [24] developed a settlement prediction model for the upper subgrade layers of ballasted tracks, which incorporated logarithmic relationships to enhance the accuracy of their predictions. Their model demonstrated that the logarithmic approach could effectively represent the complex interactions between the ballast and subgrade, leading to improved forecasting of track irregularities and settlement behavior.

The empirical settlement model proposed by Wang and Markine [25] also utilized logarithmic functions to describe the two-stage settlement process of ballast. Their research validated the model against field measurements, reinforcing the applicability of logarithmic relationships in predicting long-term settlement behavior in transition zones. This two-stage approach is critical for understanding how initial rapid settlements can transition into more gradual, logarithmic patterns over time.

In addition to these studies, the work of Hamarat *et al.* [26] highlighted the impact of unsupported sleepers on dynamic phenomena and settlement in ballasted tracks. Their findings suggested that the settlement behavior could be modeled using logarithmic functions to account for the varying degrees of support provided by the ballast, particularly in cases where sleeper support is compromised.

The influence of ballast degradation on settlement was further explored by Huang *et al.* [27], who noted that as ballast degrades, the relationship between load cycles and settlement can often be represented logarithmically. This degradation leads to increased settlement rates, which can be effectively modeled to predict future performance and maintenance needs.

Moreover, the research conducted by Sysyn *et al.* [28] on ballast consolidation under vibration loading provided additional evidence for the utility of logarithmic models in understanding settlement dynamics. Their experimental findings indicated that the consolidation behavior of ballast could be described using logarithmic relationships, which are essential for predicting how ballast will respond to repeated loading over time.

The integration of discrete element modeling (DEM) in understanding ballast behavior has also been significant. Studies such as those by Yan *et al.* [29] and Zhou *et al.* [30] employed DEM to simulate the mechanical properties of ballast under cyclic loading, revealing that the resulting settlement patterns could often be described using logarithmic functions. This approach allows for a more nuanced understanding of how ballast particles interact and deform under load, leading to settlement.

The most common and widely used logarithmic settlement deterioration model was summarized by Lichtberger [31], two models given by the ORE ("Office de Recherches et d'Essais", i.e., "Research and Testing Office" in English) from 1970 [32] and 1975 [33]. These are introduced in Section 3.2 in a more detailed manner.

2.2 Models based on power functions

The foundational work in this area often revolves around the understanding of ballast behavior under cyclic loading. For instance, Zhou *et al.* [20] conducted a comprehensive study that established a full-scale experimental facility to observe the evolution of settlement in ballasted railway tracks subjected to cyclic loading. Their findings indicated a direct correlation between the number of load cycles and the incremental settlement experienced by the track, which aligns with the principles of power functions where the response is often modeled as a function of the applied load raised to a certain exponent. This relationship is crucial for predicting long-term settlement trends in railway infrastructure.

Further investigations into the dynamic characteristics of railway ballast have been conducted using discrete element methods (DEM), which allow for a more granular analysis of ballast particle interactions. For example, Kumar *et al.* [34] explored the micro-mechanical behavior of railway ballast under cyclic loading conditions, employing DEM to simulate the effects of different ballast types and elastic layers. Their results demonstrated that the settlement behavior could be effectively modeled using power functions, particularly when considering the influence of particle size and shape on the overall performance of the ballast layer.

In addition to experimental studies, numerical modeling has played a pivotal role in understanding ballast settlement. Li *et al.* [22] developed a three-dimensional model to analyze differential railway track settlement, utilizing a cycle domain constitutive model that incorporates power function relationships to describe the stress-strain behavior of the ballast. This approach allows for the prediction of settlement patterns based on varying loading conditions and material properties, thereby enhancing the reliability of settlement forecasts in practical applications.

The impact of ballast degradation on settlement has also been a focal point of research. Xu *et al.* [35] investigated the relationship between track settlement and ballast degradation in high-speed rail systems through full-scale laboratory tests. Their findings underscored the importance of incorporating power function models to account for the nonlinear nature of ballast degradation, which significantly affects the settlement behavior of the track over time. This highlights the necessity of integrating material degradation models with power function approaches to achieve more accurate predictions of track performance.

Moreover, the influence of environmental conditions on ballast performance has been examined in various studies. Kaewunruen and Tang [21] explored the dynamic behavior of railway ballast under flooding conditions, emphasizing the need for models that can adapt to changing environmental factors. Their research suggests that power functions can be utilized to model the effects of moisture content and saturation levels on ballast stiffness and settlement, thereby providing a more comprehensive understanding of how external conditions impact railway infrastructure.

The application of power function models extends beyond just settlement predictions; they also play a crucial role in assessing the overall stability and performance of ballasted tracks. For instance, Sayeed and Shahin [36] developed numerical models to evaluate the granular layer thickness of ballasted railway track foundations, employing power functions to describe the relationship between track geometry and settlement. Their work illustrates the versatility of power function models in addressing various aspects of railway track design and maintenance.

In the context of maintenance strategies, the predictive capabilities of power function models can inform decision-making processes regarding track interventions. Farooq *et al.* [37] demonstrated that the integration of power function models into maintenance planning could lead to significant reductions in track settlement and geometry degradation rates. By quantifying the effects of different reinforcement methods on ballast performance, their findings support the notion that power functions can serve as valuable tools for optimizing railway infrastructure management.

The interplay between ballast fouling and settlement has also been a subject of investigation. Liu *et al*. [38] provided a comprehensive review of ballast fouling mechanisms and their implications for railway performance. They highlighted that power function models could effectively characterize the relationship between fouling levels and settlement, thereby aiding in the development of condition assessment criteria for railway tracks. This underscores the importance of incorporating power function approaches in evaluating the long-term effects of ballast fouling on track stability.

Furthermore, recent studies have explored the role of advanced materials in enhancing ballast performance. For example, Ižvolt *et al.* [39] investigated the application of innovative thermal insulation materials in railway track substructures. Their findings indicated that power function models could be employed to assess the impact of these materials on ballast behavior, particularly in terms of settlement reduction and thermal performance. This highlights the potential for power function models to facilitate the integration of new technologies into traditional railway systems.

The significance of power function models in the context of railway track settlement is further reinforced by their ability to accommodate various loading scenarios and material properties. For instance, Albahkali *et al.* [40] utilized finite element modeling to predict the life cycle of railway tracks, incorporating power function relationships to capture the nonlinear responses of ballast under different loading conditions. Their approach demonstrates the adaptability of power function models in addressing complex engineering challenges associated with railway infrastructure.

2.3 Models based on machine learning

One of the primary focuses of recent research is the development of predictive models for railway track degradation. Han [41] highlights the effectiveness of multiple machine learning methods, including support vector machines (SVM), artificial neural networks (ANN), and grey models (GM), in analyzing and predicting the longitudinal level of railway tracks. These models are particularly valuable as they can accommodate the random characteristics of track degradation and can yield accurate predictions even with limited data. Similarly, Liao *et al.* [42] provide a comprehensive review of various machine learning methods for predicting railway track geometry degradation, emphasizing the advantages of these techniques over traditional methods. They discuss how ML models can uncover degradation patterns and improve prediction accuracy, thus aiding in timely maintenance interventions.

Moreover, the integration of onboard monitoring systems with data science techniques has been explored by Traquinho *et al.* [43]. This study underscores the potential of AI algorithms in interpreting complex data for damage detection in/on railway tracks. The use of onboard sensors to collect real-time data, combined with machine learning algorithms, allows for proactive maintenance strategies that can significantly reduce the risk of track failures.

The importance of data-driven approaches is further supported by the work of Tsunashima [44], who utilized machine learning techniques to analyze car-body vibrations for detecting track faults. This method demonstrates how ML can be employed to extract relevant features from vibration data, thereby enhancing the reliability of condition monitoring systems. The findings indicate that machine learning can effectively classify and isolate track faults, contributing to improved maintenance practices.

In addition to predictive modeling, the application of deep learning techniques has also been explored. Sresakoolchai and Kaewunruen [45] developed a three-dimensional recurrent neural network-based model for predicting track geometry parameters. This innovative approach showcases the potential of deep learning to enhance the precision of track inspections and maintenance schedules. The authors argue that such advanced models can optimize predictive maintenance, ultimately leading to safer and more efficient railway operations.

Furthermore, the survey by Chenariyan Nakhaee *et al.* [46] consolidates various machine-learning applications in rail track maintenance, emphasizing the growing body of literature that supports the efficacy of these methods. The authors note that while machine learning has been extensively reviewed in other domains, its application in railway infrastructure remains underexplored, highlighting the need for further research in this area.

2.4 General studies

It is also worth mentioning Saussine *et al.* [47], Abadi *et al.* [48], Grossoni *et al.* [49], Alqatawna *et al.* [50] also provide a wide variety of different deterioration models in their literature review. They suggest the consideration of significantly different parameters, which is very useful in a sophisticated model and its calculation in a scientific way. However, in many cases, they cannot be used in the presence of unknown factors or approximate values of these parameters, which is significantly disadvantageous and deteriorates the accuracy of the models.

2.5 Identified research gap

Although significant progress has been made in studying the settlement behavior of ballasted railway tracks under dynamic loads, some key questions remain unanswered. One major challenge is the limited integration of degradation mechanisms, environmental factors, and maintenance strategies into existing predictive models. Most research tends to focus on isolated aspects, like the mechanical response of ballast to cyclic loading or the effects of material degradation, often overlooking how these factors interact.

While models based on logarithmic or power functions have proven effective for capturing settlement trends, their ability to adapt to a wide range of track conditions and environmental scenarios is still constrained. Emerging approaches, such as machine learning and advanced simulations, show great potential but have yet to see widespread implementation or validation in practical, real-world applications.

This study seeks to bridge these gaps by leveraging validated field data and regression analyses to refine and expand current settlement models. By integrating fresh datasets and exploring the relationships among material properties, loading patterns, and environmental influences, the research aims to create a more versatile and predictive framework for understanding track settlement.

3. Materials and methods

Section 3 is divided into two subsections: Section 3.1 covers materials, and Section 3.2 covers methods.

3.1 Materials

To avoid (unnecessary) repetition, Section 3.1 will be simplified and abbreviated in this article. The railway (trial) section, which is presented in detail in Fischer (2022a) [51] and Fischer (2022b) [52], has been examined in more detail. All the 7 subcategories/subsections have been investigated, i.e., WBS – without ballast screening, WG – without geogrid reinforcement, GG1...GG5 – reinforced by five different types of geogrids. The GG1...GG5 designations are to be seen, according to Fischer (2022a) [51].

3.2 Methods

In contrast to Fischer (2022a) [51] and Fischer (2022b) [52], it is not the track geometry measuring or classification numbers that were examined, but the precision line leveling results from 18 June 2010 to 2 October 2014 and the average settlements calculated from these. The instrument used was a Leica DNA03 standard leveling instrument, which was applied to record the relative elevation difference (i.e., the vertical position) for each measurement point relative to a fixed point in a local coordinate system every 3 sleepers (i.e., every 1.8 m). Displacement in the horizontal plane was not taken into account.

In the present analysis, not only the mean (average) but also the standard deviation of the displacements (settlements) has been taken into account.

The measurements and track geometry corrections were made during this time interval, as shown in Table 1, the data and details of which (e.g., the calculated through-rolled axle tons for each measurement time) are given in Tables 1 and Table 2.

Table 1
Data of measurements and tampings – 1 (the bold rows were considered in the following calculations as tampings) [51]

No. of days (June 17, 2010, is Day #0)	Measurement (geodetic levelling)	Geometry correction and tamping executed by mechanized maintenance train (MMT)	Through-rolled axle tons [MGT]
0	no	yes	0.000
1	yes	no	0.019
8	yes	no	0.178
34	no	yes	0.727
42	yes	no	0.895
95	no	yes	1.991
119	yes	no	2.487
124	**no**	**yes**	**2.592**
126	yes	no	2.634
284	no	yes	5.776
319	yes	no	6.471
356	no	yes	7.163
487	yes	no	9.610
490	no	yes	9.680

Table 2
Data of measurements and tampings – 2 (the bold rows were considered in the following calculations as tampings) [51]

No. of days (June 17, 2010, is Day #0)	Measurement (geodetic levelling)	Geometry correction and tamping executed by mechanized maintenance train (MMT)	Through-rolled axle tons [MGT]
491	**yes**	**no**	**9.703**
504	no	yes	9.928
863	no	yes	16.874
872	**yes**	**no**	**17.047**
874	yes	no	17.086
1020	no	yes	19.906
1203	yes	no	22.355
1210	no	yes	23.769
1568	yes	no	33.621

It should be noted that an intervention was individually agreed with the MÁV (Hungarian State Railways) Track Maintenance Department, Győr, Hungary and that on days #124, #490 and #872, interventions (geometrical correction, i.e., tampings) were made that could have significantly modified the longitudinal level of the track and the compacted crushed stone ballast beam.

Therefore, only these three were considered (see Tables 1-2). In each case, a directional fault (i.e., track alignment fault) reduction procedure was applied, while a longitudinal fault (i.e., longitudinal level fault) elimination procedure was applied. The tamping machines were Plasser 08 and 09 series.

It should be mentioned that each deterioration (settlement) function has been analyzed in such a way that the first measurements after the geometrical corrections (tampings) have been omitted. The observed deterioration trend still "appears" to be continuous. Their details are introduced and explained in Section 4.

Based on Section 2.1, the deterioration (settlement) functions considered in the 1970 [32] and 1975 [33] ORE studies are as follows, see Eqs. (1-3) and Tables 3 and 4. The formulae and tables are taken from Lichtberger [31].

$$e_N = e_1 \cdot (1 + b \cdot log_{10} N) \tag{1}$$

Here, e_N is the settlement after N load alternations [mm] [31,32], N is the number of load alternations [piece] [31,32], b is a constant (~0.2 for an individual sleeper, ~0.43 for the track grid) [-] [31,32], e_1 is the settlement after the first load alternation [mm] [31,32].

$$e_T = a_1 + a_0 \cdot log_{10}\left(\frac{T}{2}\right) \tag{2}$$

Here, e_T repersents the settlement after an operational load T in [MGT] [31,33], while a_0 and a_1 are the coefficients [-] that should be determined and applied based on Table 3 [31,33].

Table 3
Ranges of the coefficient of track settlement [31,33]

Designation	a_0 [-]	a_1 [-]
high-quality track	2…4	6…10
average track	4…6	10…15
poor track	6…10	15…20

$$\sigma_T = b_1 + b_0 \cdot log_{10}\left(\frac{T}{2}\right) \tag{3}$$

Here, σ_T represents the standard deviation of the settlement after an operational load T in [MGT] [31,33], while b_0 and b_1 are the coefficients [-] that should be determined and applied based on Table 4 [31,33].

Table 4
Ranges of standard deviations of various track qualities [31,33]

Designation	b_0 [-]	b_1 [-]
high-quality track	0.02…0.5	0.5…1
average track	0.5…0.8	1…1.5
poor track	0.8…2	1.5…5

Due to the fact that MS Excel software uses the Euler number (e=2.718...) based logarithm (i.e., ln) instead of the logarithm based on 10 (i.e., $\log_{10}$), the formulas in Eqs. (2-3) have been modified; see. Eqs. (4-9).

$$e_T = a_2 + a_3 \cdot ln\left(\frac{T}{2}\right) \tag{4}$$

$$a_0 = a_2 \cdot ln(10) \tag{5}$$
$$a_1 = a_2 \cdot ln(10) \cdot log_{10}(2) + a_3 \tag{6}$$

Here a_2 and a_3 are the coefficients [-] to substitute a_0 and a_1.

$$\sigma_T = b_2 + b_3 \cdot ln\left(\frac{T}{2}\right) \tag{7}$$
$$b_0 = b_2 \cdot ln(10) \tag{8}$$
$$b_1 = b_2 \cdot ln(10) \cdot log_{10}(2) + b_3 \tag{9}$$

Here b_2 and b_3 are the coefficients [-] to substitute b_0 and b_1.

Of course, the use of Eqs. (4-9) is not mandatory, but it makes the evaluation much easier to use.

4. Results and discussion

Figure 1 shows the average settlement values calculated from the leveling data for the sections marked WG.

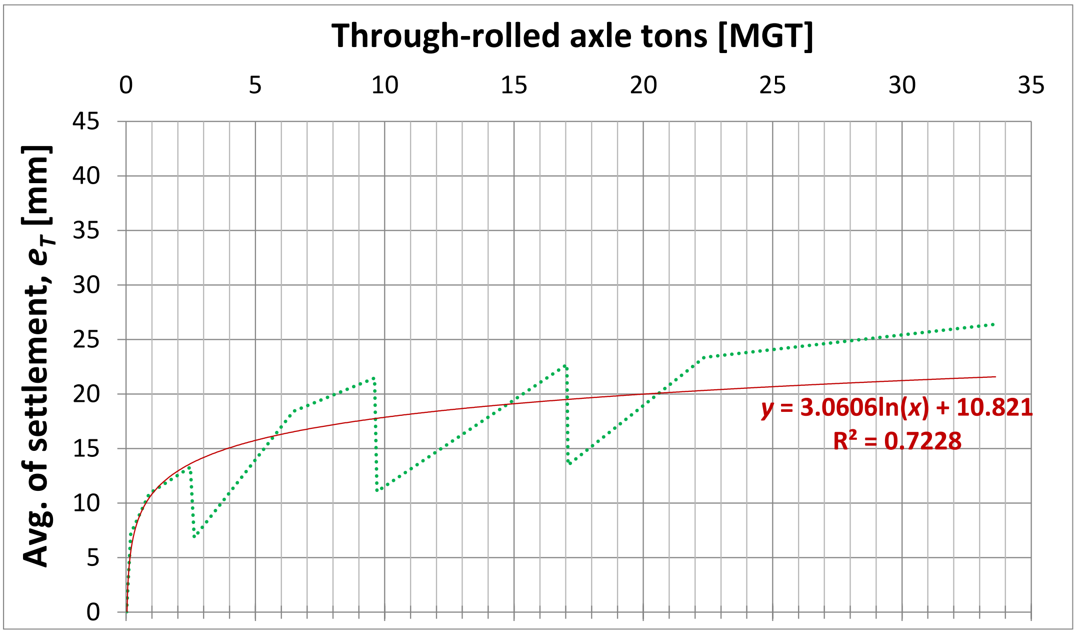

Fig. 1. Average of settlement as a function of through-rolled axle tons in the case of all measurements; the considered section is the WG

As mentioned in Section 3.2, the leveling values of day #126, day #491 and day #874 and the average settlement values calculated from them have been neglected because they do not affect the deterioration trend shown in Figure 1 (in a significant way). The function without them is shown in Figure 2.

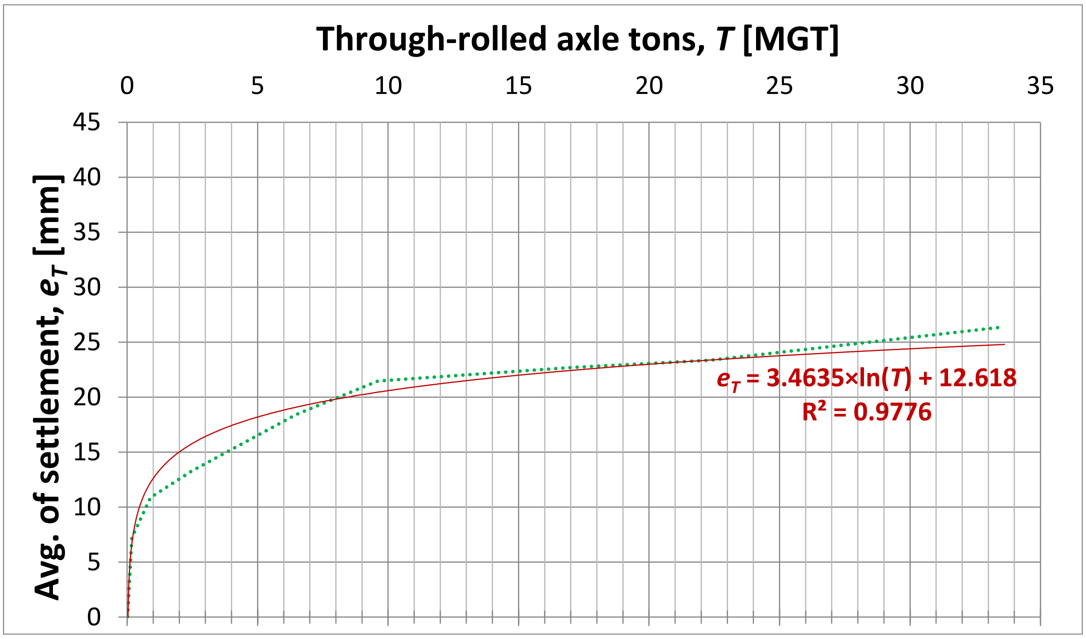

Fig. 2. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the WG

Of course, in Figure 1 and Figure 2, the calculated logarithmic deterioration regression function equation changed, but it can be seen that the regression function calculated without the measurement results of the given days gave a better approximation of the observed trend.

Figures 3-9 show the average settlement values by subsection (WG, WBS and GG1...GG5), but three additional curves are shown on each graph. These should be interpreted according to Eqs. (4-6). The values given for the regression functions are the coefficients a_2 and a_3.

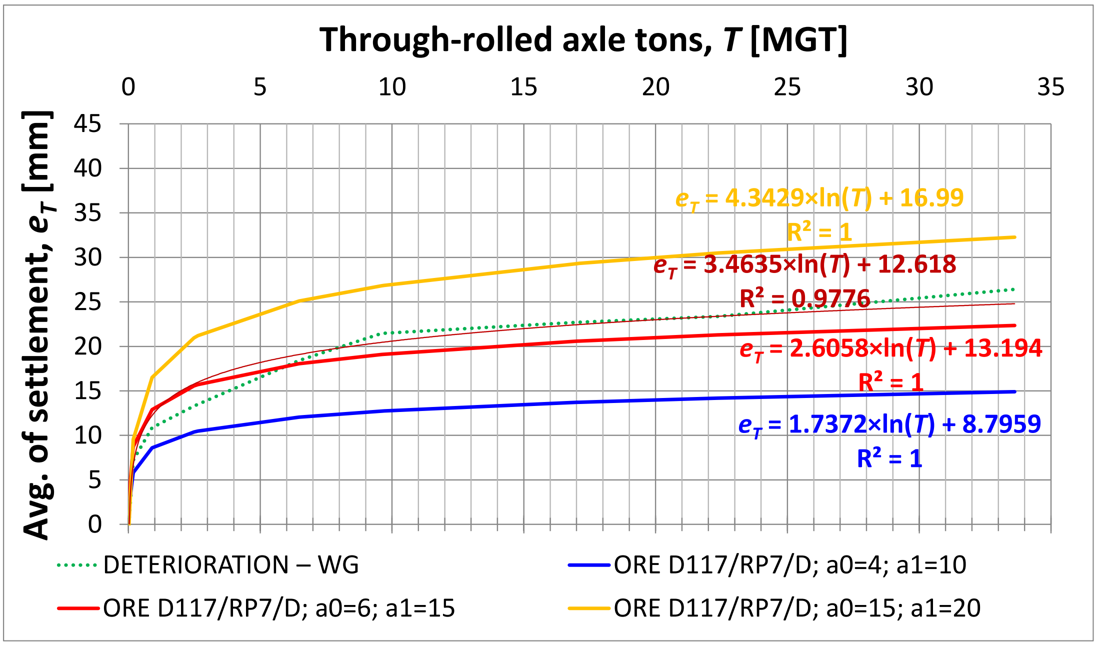

Fig. 3. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the WG. Three standard curves from [33] are also illustrated

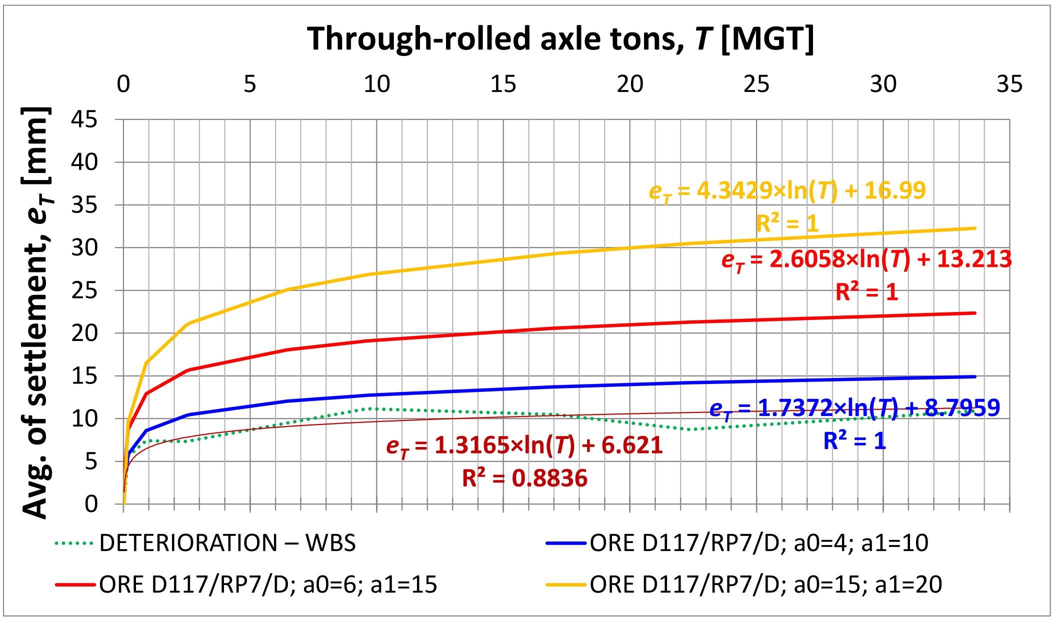

Fig. 4. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the WBS. Three standard curves from [33] are also illustrated

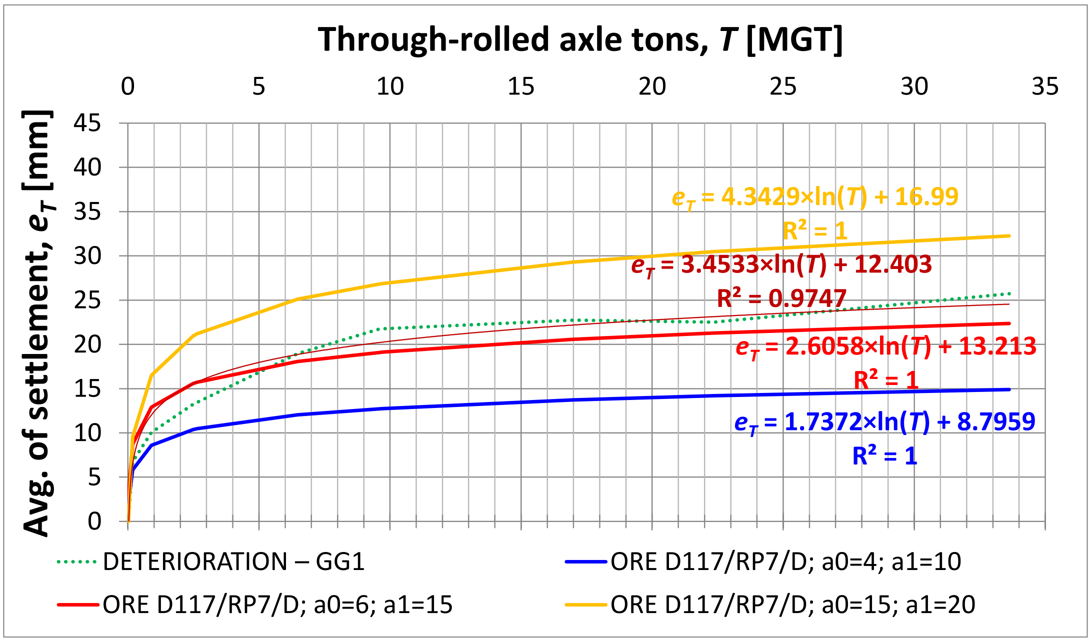

Fig. 5. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG1. Three standard curves from [33] are also illustrated

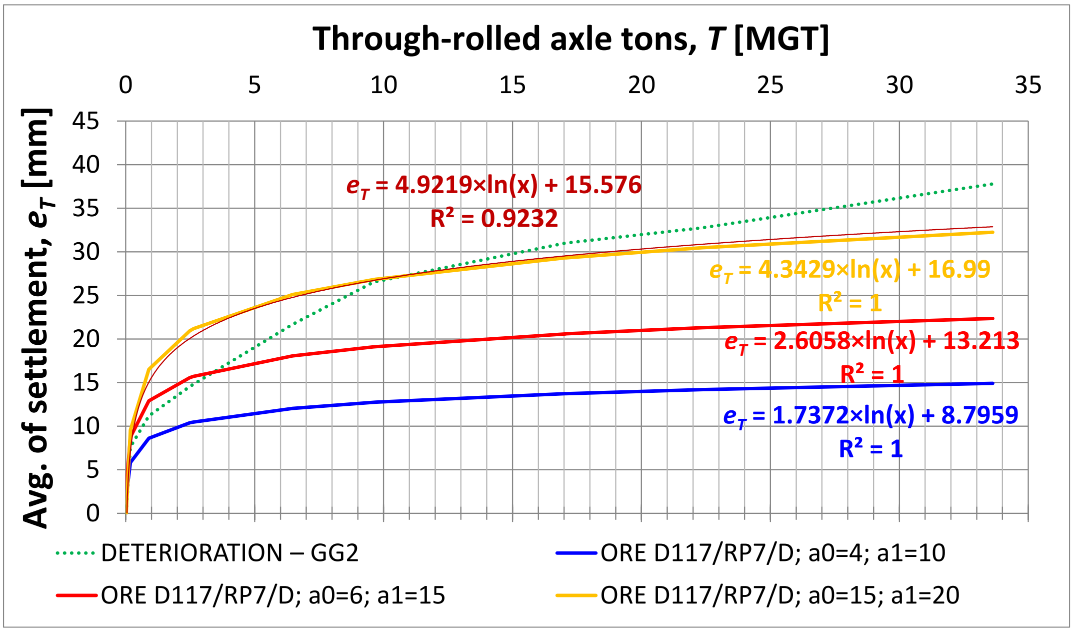

Fig. 6. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG2. Three standard curves from [33] are also illustrated

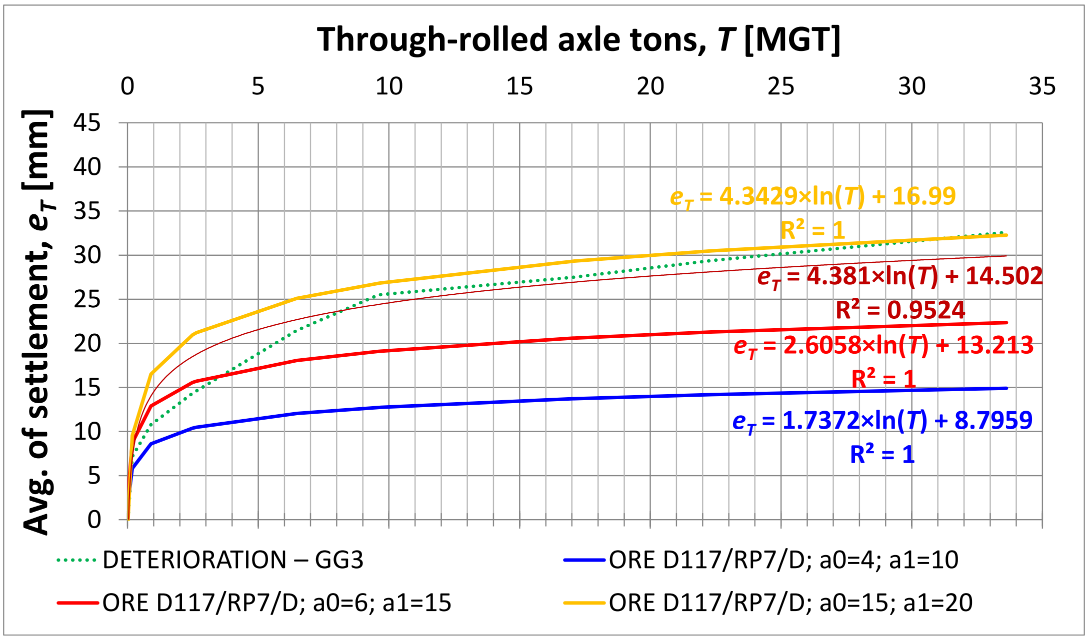

Fig. 7. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG3. Three standard curves from [33] are also illustrated

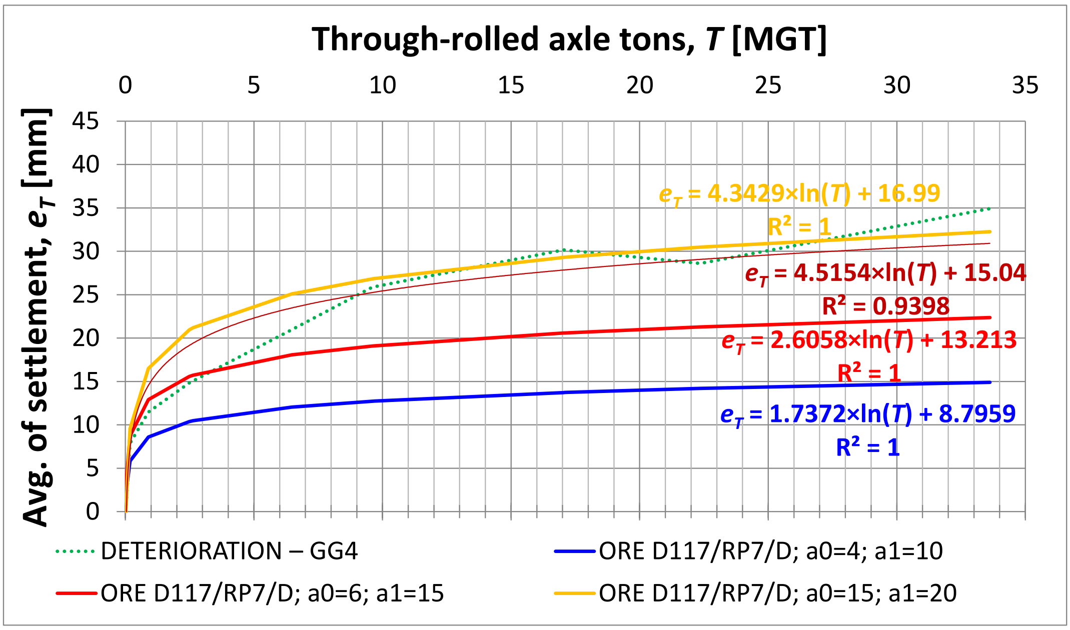

Fig. 8. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG4. Three standard curves from [33] are also illustrated

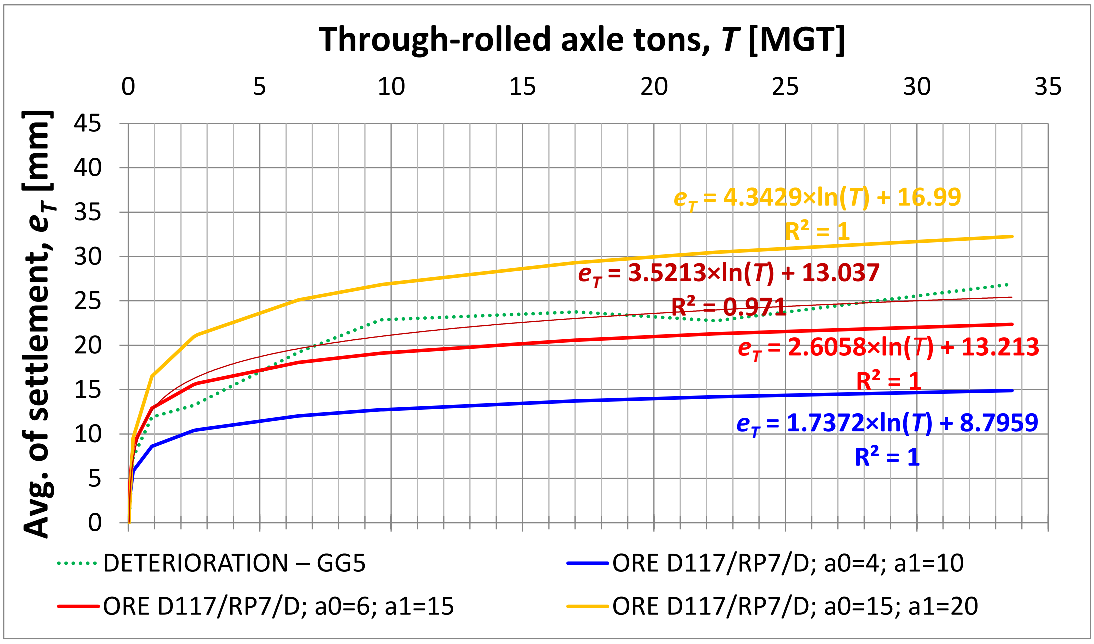

Fig. 9. Average of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG5. Three standard curves from [33] are also illustrated

Figure 10 shows the standard deviation of the settlement values calculated from the leveling data for the sections marked WG.

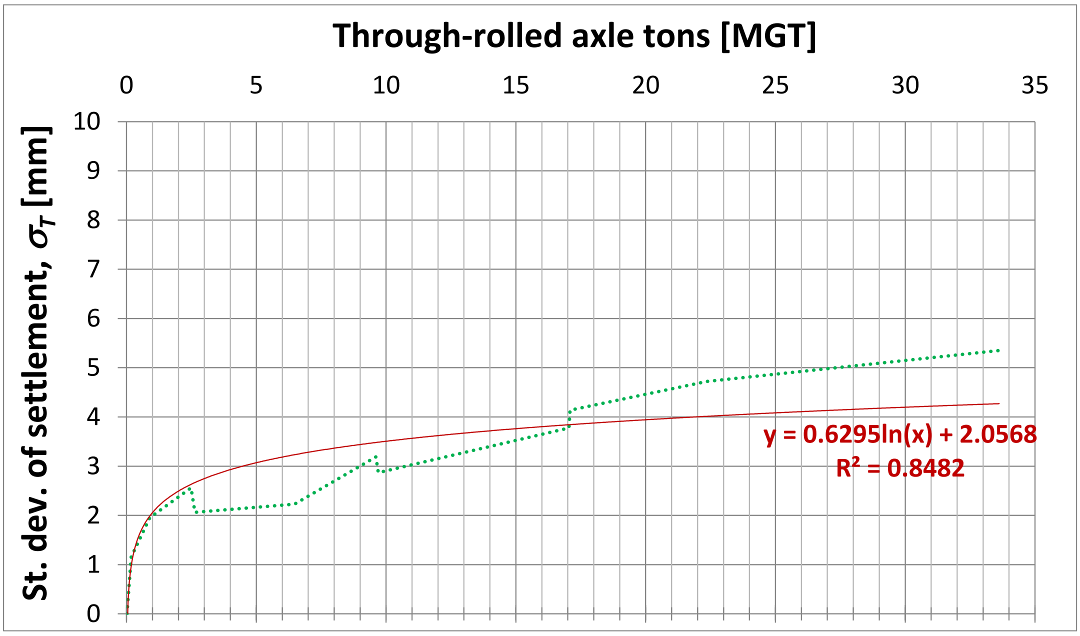

Fig. 10. Standard deviation of settlement as a function of through-rolled axle tons in the case of all measurements; the considered section is the WG

As in Figures 1 and 2, the standard deviations of the calculated settlement values have been neglected because they do not significantly affect the trend of deterioration shown in Figure 10. The function without them is shown in Figure 11.

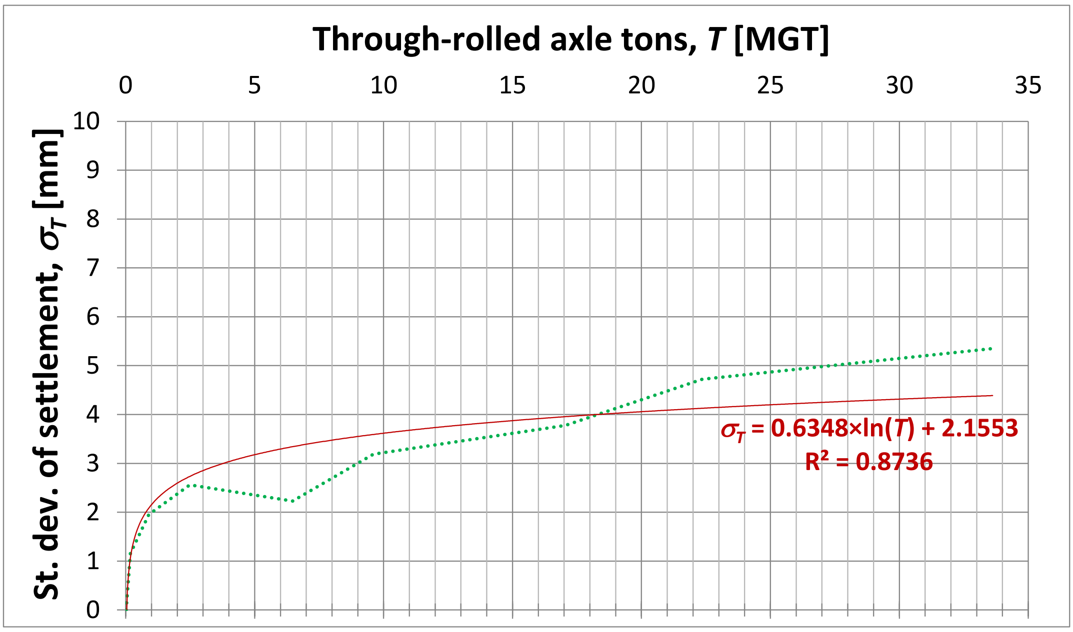

Fig. 11. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the WG

Of course, in Figure 10 and Figure 11, the calculated logarithmic deterioration regression function equation changed, but it can be seen that the regression function calculated without the measurement results of the given days gave a better approximation of the observed trend.

Figures 12-18 show the calculated standard deviation of the settlement values by subsection (WG, WBS and GG1...GG5), but three additional curves are shown on each graph. These should be interpreted according to Eqs. (7-9). The values given for the regression functions are the coefficients b_2 and b_3.

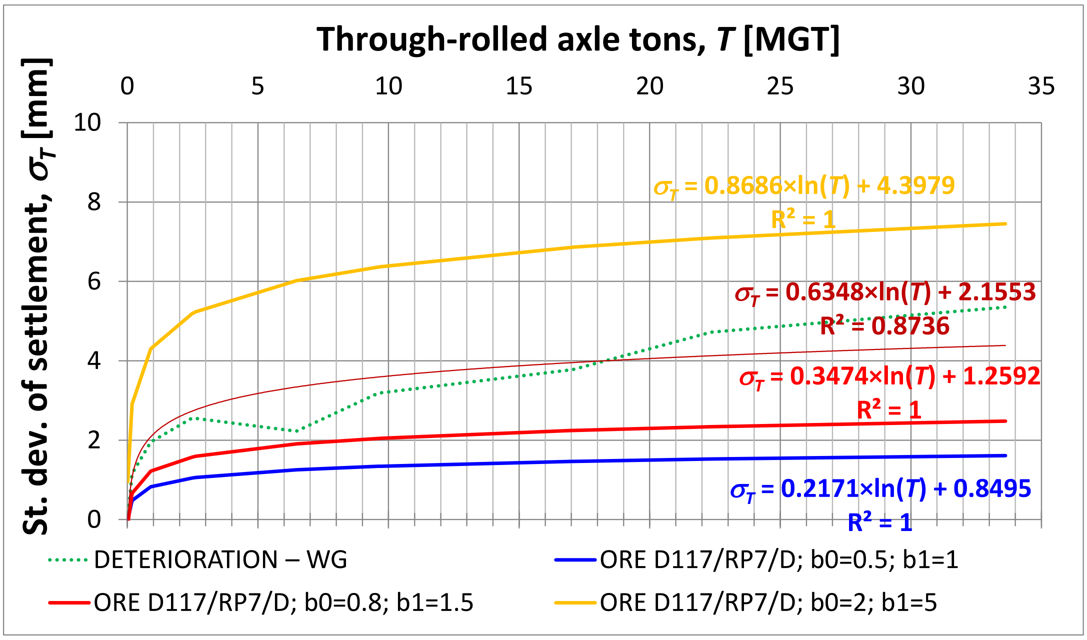

Fig. 12. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the WG. Three standard curves from [33] are also illustrated

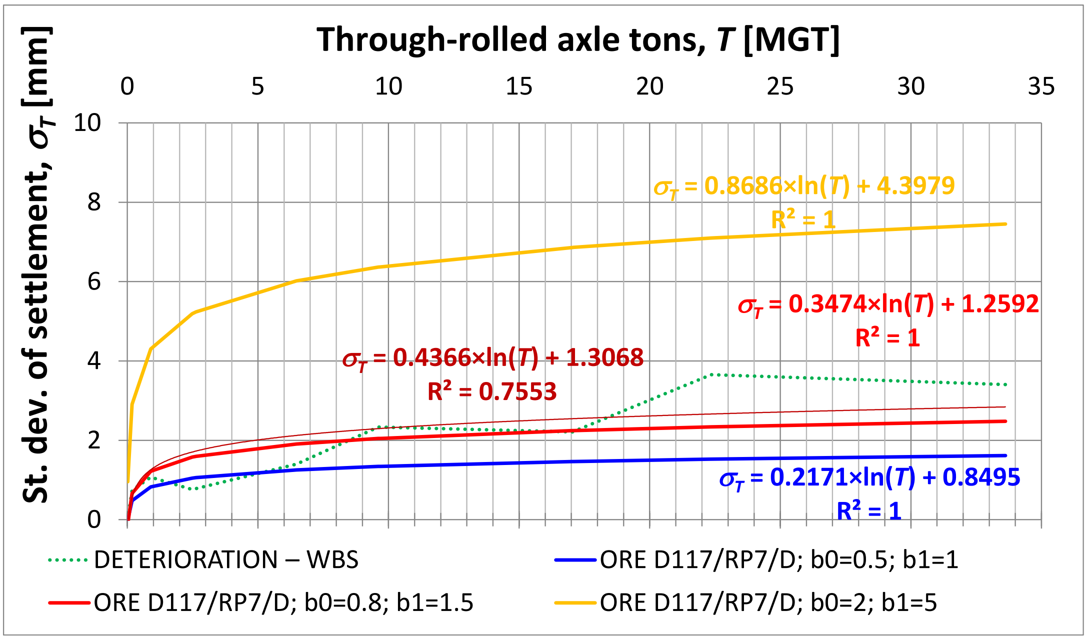

Fig. 13. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the WBS. Three standard curves from [33] are also illustrated

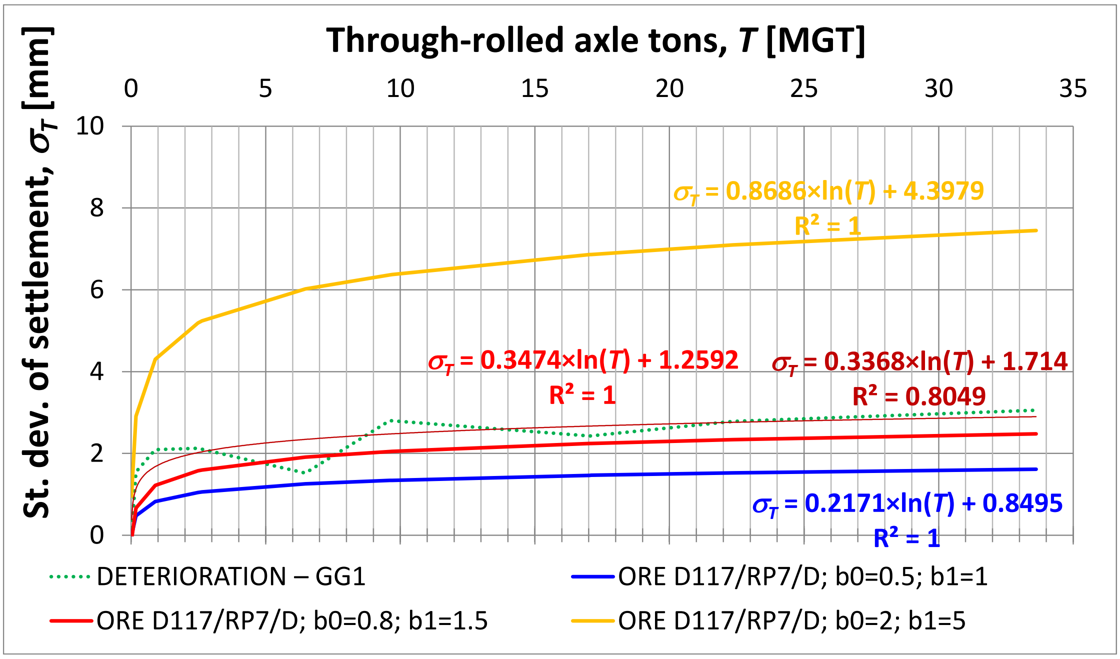

Fig. 14. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG1. Three standard curves from [33] are also illustrated

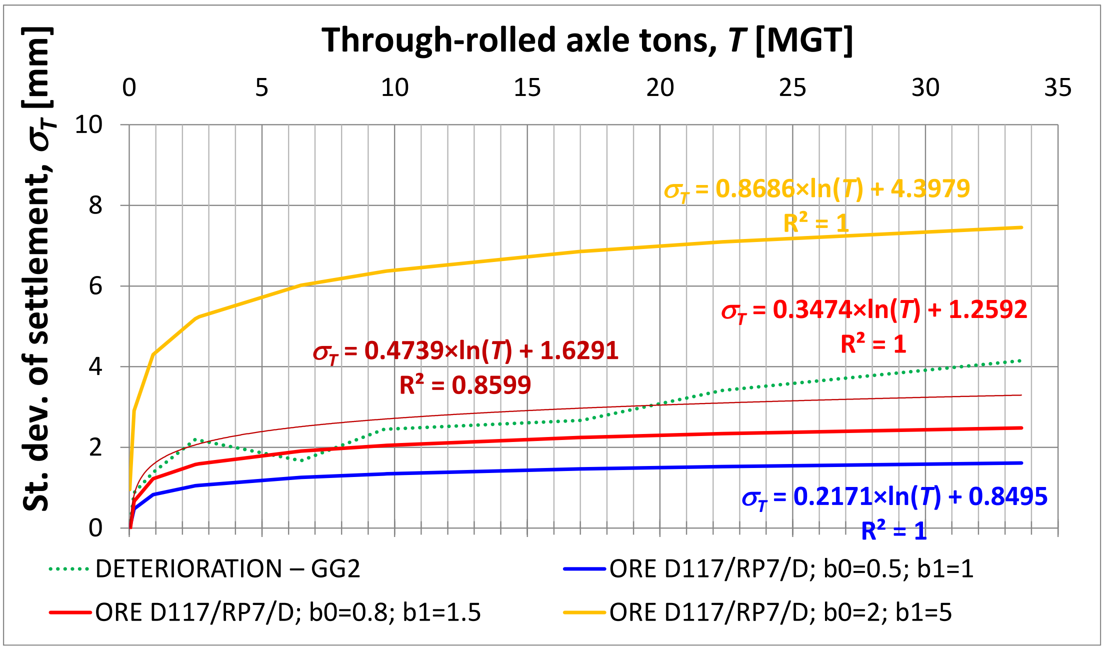

Fig. 15. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG2. Three standard curves from [33] are also illustrated

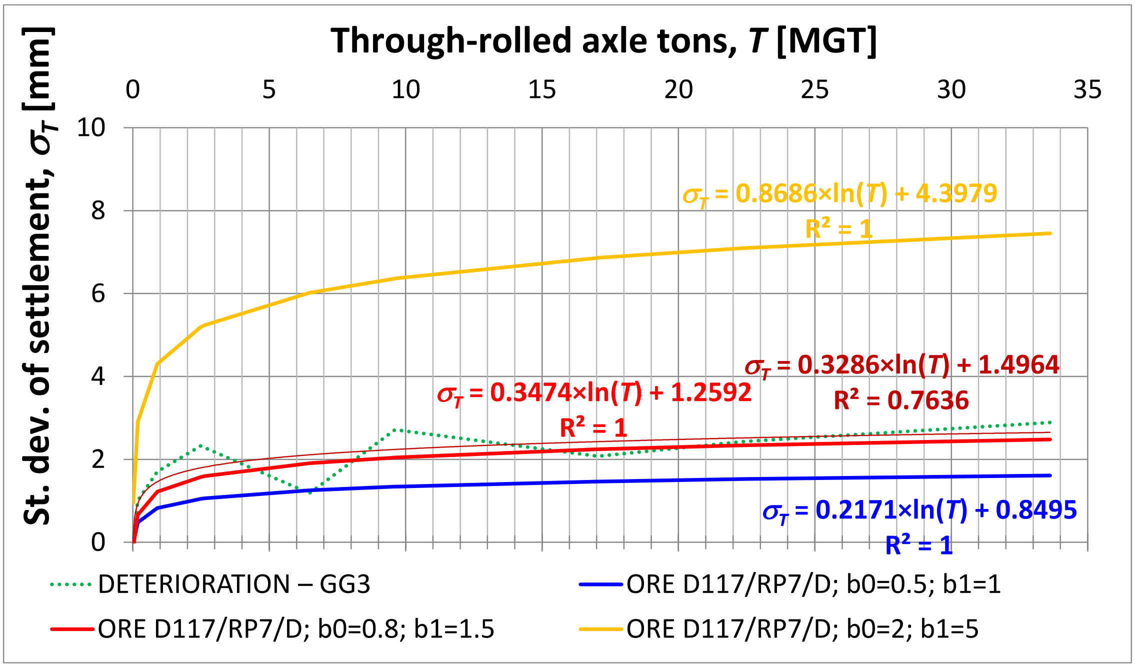

Fig. 16. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG3. Three standard curves from [33] are also illustrated

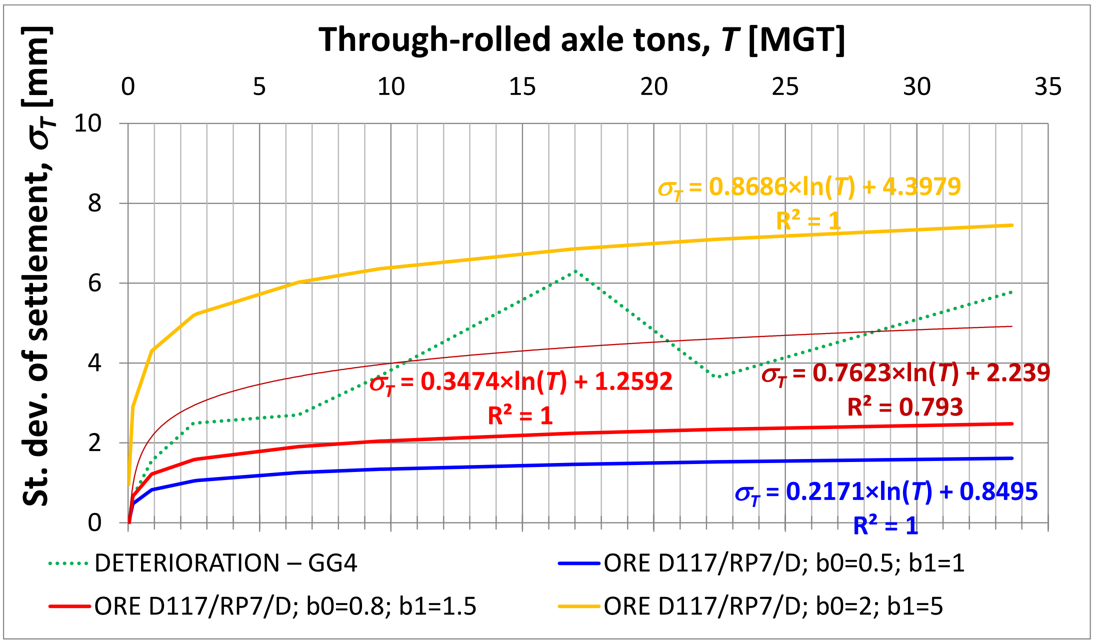

Fig. 17. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG4. Three standard curves from [33] are also illustrated

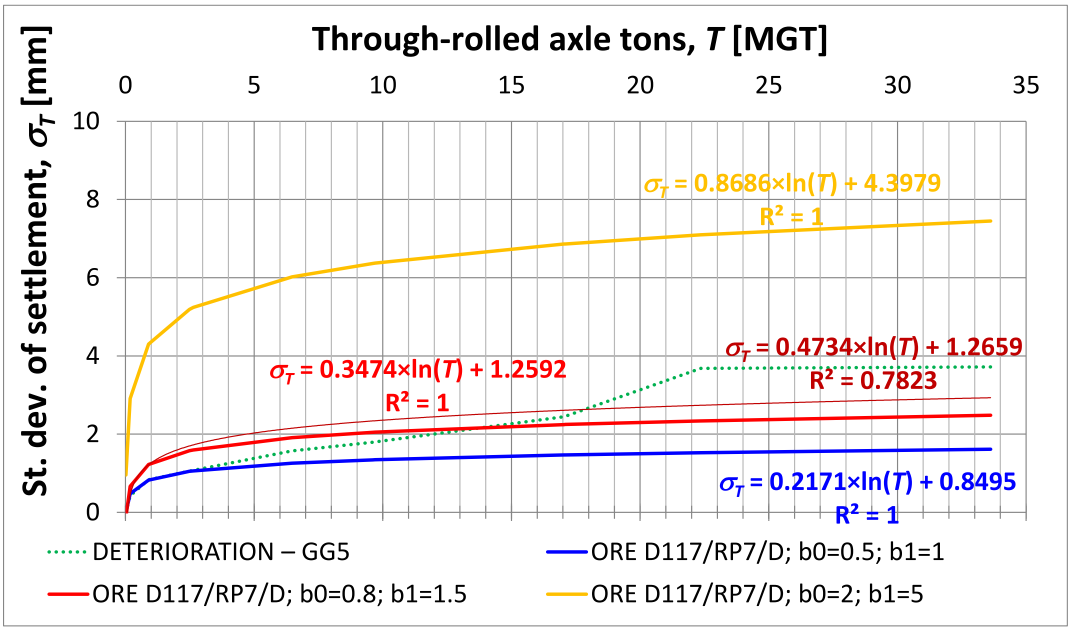

Fig. 18. Standard deviation of settlement as a function of through-rolled axle tons in the case of neglecting three measurements; the considered section is the GG5. Three standard curves from [33] are also illustrated

From the diagrams in Figures 3-9 and Figures 12-18, the coefficients of each subsection a_2, a_3, b_2 and b_3 and their relationships to the three-to-three standard curves can be read clearly and precisely.

It should be noted that this paper was not intended to estimate the deterioration functions and/or amplification effects given in Fischer (2022a) [51] and Fischer (2022b) [52], but they were similar to those in [51]. That is, for the e_T feature, WBS, GG1 and GG5 were the most favorable, while for the σ_T parameter, GG1, GG2 and GG3 were the most advantageous reinforcements.

Based on Figures 3-9 and Figures 12-18, the following additional results can be drawn:

- it should be pointed out once again that measurements after the mechanized maintenance train or other track geometry correction have not been taken into account (see Section 3.2),
- the ORE 1970 [32] and ORE 1975 [33] models given in Lichtberger's book [31] have been used as the most widely used deterioration models for comparison,
- the above was done in order to verify their practical use and usability and, if verified, to give the decay function of each of the differently designed test sections,
- - because of the use of MS Excel, the regression functions were not given in terms of the logarithm of 10 (i.e., $\log_{10}$). However, in terms of the logarithm of the natural number (*e*) (i.e., ln), with the conversion formulae given,
- not only the mean of the settlement but also the standard distribution characteristic has been analyzed; for this purpose, the models published in the ORE 1970 [32] and the ORE 1975 [33] studies and recommendations have been used for comparison,
- it has been shown that the ORE 1970 [32] and ORE 1975 [33] models are adequate but that the constants a_0, a_1, b_0 and b_1 need to be refined. The given intervals are approximately suitable for an approximate estimation,
- the deterioration of the sections without ballast screening (WBS) was significantly lower but could be predicted,
- the comparison with the WBS is not entirely correct because, in this case, a very well "settled" longitudinal level has already been established, but the result is suitable to give an estimate or prognosis for such cases-situations as well,

- the R^2 values of the determined regression functions are higher than 0.88 in the case of e_T and 0.75 in the case of σ_T,
- in international literature research, the best and simplest models to use are those in which the developing settlement can be determined solely on the basis of the through-rolled axle tons. In contrast, the sophisticated models take more into account (e.g., static value of axle load, allowed speed on the track and actual-applied vehicle speeds, value of vertical compressive stress under the sleeper and value/ratio of σ_1 and σ_3 (i.e., the main stresses), material and PSD of the crushed stone ballast used, density-porosity and thickness of the ballast, vertical plastic settlement due to the passage of the first axle, etc.)

5. Conclusions

Based on the results detailed in Section 4, the following conclusions can be drawn:

- Validation of models: the main objective was to validate the proven settlement and deterioration models, prioritizing comparisons with the ORE 1970 [32] and 1975 [33] studies. These models, with their constants and conversion factors (e.g., a_0, a_1, b_0, b_1), enable easy categorization and forecasting of settlement functions. However, these predictions are subject to strong restrictions and require further refinement for precise use.
- Role of track geometry corrections: it was demonstrated that the first measurement after track geometry corrections could be neglected without significantly affecting the regression function. However, this approach must be applied cautiously, as its validity depends on the specific conditions of the compacted ballast beam and geometry correction methods. It must be certified with other case studies.
- Critical compactness of ballast bed: a significant challenge lies in understanding and controlling the compactness (density) of the ballast bed. The results underscore its importance in settlement behavior, highlighting the need for methods to measure and maintain this critical parameter effectively during and after maintenance operations.
- Practical application of deterioration functions: international literature and this study both emphasize that simpler models, which correlate settlement solely with through-rolled axle tons, are easier to apply in practice. However, more sophisticated models, which account for parameters such as axle load, vehicle speeds, ballast material properties, and stress conditions, offer greater predictive accuracy but require extensive data.
- Refinement of constants for improved predictions: the constants (a_0, a_1, b_0, b_1) in the ORE models [31-33] need refinement to accommodate contemporary railway conditions. While the existing intervals provide approximate estimates, further studies are necessary to calibrate these values for different track qualities.
- Integration of experimental and simulation studies: supplementary modeling using laboratory tests and discrete element simulations is crucial for enhancing the predictive power of settlement models. This integration can bridge the gap between theoretical models and real-world applications.
- Implications for maintenance and design: the findings have practical implications for maintenance strategies, particularly in predicting settlement trends and planning interventions. The study demonstrates that sections without ballast screening exhibited significantly lower settlement, albeit under specific conditions. This insight provides a basis for optimizing ballast reinforcement and track design.
- Future research possibilities: the geometric distortion (deterioration) of the railway track could even be measured by DIC (digital image correlation) [53], which could provide a relevant

innovation in the field of railway track (especially in civil engineering); it may can be supplemented by (FEM) finite element modeling or another computer simulation [54].

By refining deterioration models and integrating advanced methodologies, this research lays the groundwork for improving the reliability and durability of railway track systems.

Author Contributions

Author have read and agreed to the published version of the manuscript.

Data Availability Statement

The data supporting the reported results was obtained through numerical simulations performed by the authors on the cluster of the Politehnica University Timisoara, Romania.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This research was not funded by any grant.

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