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A Fermatean Fuzzy Decision-Making Model for Manufacturing Outsourcing Vendor Selection: An Improved Combined Compromise Solution Method

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ABSTRACT

Selecting the most suitable manufacturing outsource vendors (MOVs) is crucial, as these decisions significantly influence organizational performance, environmental sustainability, and resilience to disruptions. However, the evaluation process often involves subjective judgments, which can introduce uncertainty and inconsistency. To address this problem, this study adopts the improved Combined Compromise Solution (CoCoSo) method, enhanced with Fermatean fuzzy logic, as a consensus-driven decision-making framework. To further improve accuracy, Fermatean fuzzy Generalized Dombi weighted aggregation operators are employed. Criteria weights are determined using dispersion measures and cross-entropy techniques. The application of the proposed method is demonstrated through a case study involving a precision gearbox manufacturing company. Among the evaluated criteria, carbon emission reduction, waste recycling rate, and on-time delivery rate are identified as the most critical, with respective weights of 0.3160, 0.1254, and 0.1254. Of the four candidate MOVs (A1, A2, A3, and A4), vendor A4 is ranked highest with a utility score of 1.96145. A comparative analysis confirms the superior performance of the proposed approach compared to alternative decision-making models.

1. Introduction

The Manufacturing outsourcing (MO) has emerged as a key tactic for businesses looking to cut expenses, improve operational effectiveness, and focus on their core competencies. MO mostly describes the practice of businesses assigning all or a portion of their manufacturing processes to outside vendors, frequently located in different regions [1]. These suppliers, referred to as MO vendors (MOVs), offer specialized services that assist businesses in cutting expenses and increasing productivity. MOVs are essential to global supply chain management (SCM) because they allow businesses to produce things on their behalf and expedite their production processes. Organizations

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can focus their resources on marketing, customer service, and innovation initiatives that directly enhance their competitive edge by outsourcing production duties to specialized vendors [2]. Several studies have emphasized how important vendor selection is in determining the dynamics of SCM and how it is carried out [3]. There is a lot of weight in the vendor selection process since choosing the wrong vendor can negatively impact both financial performance and operational efficiency. On the other hand, selecting the correct vendor can result in a number of benefits like lower procurement costs, increased end-user satisfaction, and a stronger competitive advantage in the market [4].

Despite the significant advantages it offers in enhancing crucial elements like delivery, quality, flexibility, cost savings, service levels, and innovation, many firms find it difficult to establish a trustworthy vendor selection process. With the global expansion of outsourcing, buyers must carefully evaluate and select their providers to ensure reliable delivery of goods and services. Failure of a provider to meet expectations can cause significant disruption to operations and lead to substantial financial losses. A comprehensive evaluation and selection process is essential to ensure the effective performance of MOVs. This approach should incorporate suitable mathematical methods for rigorous analysis and align evaluation criteria with the specific needs of the organization [5].

The conceptualization of fuzzy sets (FSs) emerged as a response to the need for accommodating imprecise human judgments when addressing real-world problems [6]. Yager [7] introduced Pythagorean FS (PFS), an enhanced iteration of FSs characterized by the inclusion of membership value (MV) and non-MV (NMV). PFSs adhere to the constraint that the sum of the squares of MV and NMV must be equal to or less than 1. PFSs don't deal with the situation when the sum of MV^2 and MNV^2 is > 1. Senapati and Yager [8] introduced Fermatean FSs (FFSs) to handle such situations. In FFSs, the sum of $MV^3$and MNV^3 is ≤ 1. FFSs demonstrate superior potency and competence in effectively addressing uncertain multi-criteria decision-making (MCDM) problems. To integrate information within FFSs, FF weighted operators [9], FF Dombi weighted operators [10], FF Hamacher weighted operators [11], and FF Einstein operators [12] have been used so far. The existing FF decision support models can be categorized into two main groups including (i) compromise solution-based models, which include methods like technique for order of preference by similarity to ideal solution (TOPSIS) [8] and multi-objective optimization by ratio analysis plus full multiplicative form (MULTIMOORA) [12]; and (ii) utility-based models, which encompass methods like VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) [13] and Measurement of Alternatives and Ranking according to COmpromise Solution (MARCOS) [14]. On the other hand, FFS based models have many applications [12-22].

MCDM methods are very effective for handling imprecise information [23, 24]. Numerous MCDM methods like TOPSIS, VIKOR, COmplex PRoportional Assessment (COPRAS), Evaluation based on Distance from Average Solution (EDAS), Weighted Aggregated Sum Product Assessment (WASPAS) and MULTIMOORA have been introduced to handle uncertain data. The application of these methods to decision-making problems can lead to significant alterations in the ranking results, depending on changes in the weight distribution of the criteria. Yazdani *et al.* [25] introduced Combined Compromise Solution method (CoCoSo) to tackle this deficiency. CoCoSo method combines weighted sum model (WSM) and weighted product model (WPM). This method has been employed for assessing medical logistics suppliers [26], evaluating 5G industries [27], supplier choice in building management [28], evaluation of financial risks of companies [29], assessment of recycling partners [30], technology assessment for the treatment of medical waste [31], sustainable reverse logistics service provider selection [16]; assessment of the barriers of Internet of Things (IoT) adoption for waste treatment in smart cities [32], stock portfolio selection [33], blockchain platform evaluation

[34], power battery recycling technology selection [35], group decision-making [36], unmanned aerial vehicles [37], and evaluation of decision-support tools for livestock farming [38]. The main drawback of CoCoSo is the non-inclusion of ideal and anti-ideal solutions.

However, an FF decision aid framework has not yet been employed to select MOVs. To address this gap, employing advance decision-making tools like aggregation operators (AOs) and FF decision-making approaches becomes essential. These tools have demonstrated effectiveness in consolidating input data for decision-making across various domains but often lack the flexibility to adapt to changing risk preferences in dynamic environments like metaverse. The challenge is further intensified by diverse perspectives of experts involved in the selection process, imposing a robust consensus-reaching process to ensure rational decision-making. The consolidation of all input data into a unified entity is effectively facilitated by the use of AOs. These AOs have proven to be highly valuable in various applications, including decision support, data analysis, information processing, shape recognition, and applications in neural networks. The existing FF AOs [9-12] are deficient in providing robust adaptability to evolving risk preferences within the FF environment. Besides, FFS-based existing tools do not address the crucial aspect of the “consensus-reaching process among experts”. Professionals with diverse backgrounds and experiences engaged in group decision-making often hold views that significantly differ from one another. Consequently, there is a pressing need for a consensus-building procedure among practitioners to enhance agreement levels [39]. Despite the introduction of several consensus frameworks in prior research [40-43], none of them have been developed specifically with FFSs for selecting MOVs.

The problems mentioned above served as motivation for the development of a consensus-based FF-improved CoCoSo model. The contributions are as follows:

i. A pragmatic decision support model is provided to assess and prioritize MOVs.
ii. A new generalized and resilient weighted AOs are proven by employing the characteristics of Generalized Dombi (GD) operations to address decision-making problems with FF information.
iii. A consensus-reaching process is developed with GD-weighted AOs.
iv. An improved version of CoCoSo method is developed under FF context by imposing IDS, AIDS, and UDs.
v. Sensitivity analysis of criteria weights is performed.
vi. A comprehensive comparative study between the proposed and established models is provided.

The following section (Section 2) addresses essential concepts of the study. It also presents GD operations along with the corresponding weighted AOs. Lastly, it introduces the proposed FF-improved CoCoSo method with consensus reaching. Section 3 presents a case study of MOV selection using this method, while Section 4 discusses results, sensitivity, and comparative analyses. Finally, Section 5 summarizes findings, highlights uniqueness, contributions, and also suggests future research directions.

2. Methodology

2.1 Basic concepts

Definition 1 [8]: An FFS ζ on Γ is described by $\zeta = \{\langle y_i, \mu(y_i), \nu(y_i)\rangle | y_i \in \Gamma\}$, where $\mu, \nu : \Gamma \to [0,1]$ are the BG and NBG of $y_i \in \Gamma$ to ζ, respectively, satisfying $0 \le (\mu(y_i))^3 + (\nu(y_i))^3 \le 1$. Also, we use $\pi_\zeta(y_i) = \sqrt[3]{1-(\mu_\zeta(y_i))^3 - (\nu_\zeta(y_i))^3}$. $\zeta = <\mu, \nu>$ is used to denote a FF number (FFN) if Γ comprises a single object where $\mu, \nu \in [0,1]$ and $0 \le \mu^3 + \nu^3 \le 1$.

Definition 2 [41]: For a FFN $\zeta = <\mu, \nu>$,

$$s(\zeta) = \frac{1}{2}(1 + \mu^3 - \nu^3) \tag{1}$$

is depicted as the "score" of ζ. For the FFNs $\zeta_1 = <\mu_1, \nu_1>$ and $\zeta_2 = <\mu_2, \nu_2>$, a preferencing procedure is: $\zeta_1 \succ \zeta_2$ if $s(\zeta_1) > s(\zeta_2)$.

Definition 3 [41]: The basic operations between FFNs $\zeta_1 = <\mu_1, \nu_1>$ and $\zeta_2 = <\mu_2, \nu_2>$ are:

(*i*) $\zeta_1^c = <\nu_1, \mu_1>$,

(*ii*) $\zeta_1 \oplus \zeta_2 = \left\langle \sqrt[3]{\mu_1^3 + \mu_2^3 - \mu_1^3\mu_2^3}, \nu_1\nu_2 \right\rangle$,

(*iii*) $\zeta_1 \otimes \zeta_2 = \left\langle \mu_1\mu_2, \sqrt[3]{\nu_1^3 + \nu_2^3 - \nu_1^3\nu_2^3} \right\rangle$,

(*iv*) $\lambda\zeta_1 = \left\langle \sqrt[3]{1 - \left(1 - \mu_1^3\right)^\lambda}, \nu_1^\lambda \right\rangle \ (\lambda > 0)$,

(*v*) $\zeta_1^\lambda = \left\langle \mu_1^\lambda, \sqrt[3]{1 - \left(1 - \nu_1^3\right)^\lambda} \right\rangle \ (\lambda > 0)$.

Definition 4 [16]: The distance between FFNs $\zeta_1 = <\mu_1, \nu_1>$ and $\zeta_2 = <\mu_2, \nu_2>$ is estimated as follows:

$$D(\zeta_1, \zeta_2) = \sqrt{0.5 \times \left((\mu_1^3 - \mu_2^3)^2 + (\nu_1^3 - \nu_2^3)^2 + (\pi_1^3 - \pi_2^3)^2\right)}. \tag{2}$$

Definition 5 [41]: The FFWA and FFWG operators are presented by

$$FFWA(\zeta_1, \zeta_2, ..., \zeta_n) = \overset{n}{\underset{j=1}{\oplus}} w_j \zeta_j = \left\langle \sqrt[3]{1 - \prod_{j=1}^{n} (1 - \mu_j{}^3)^{w_j}}, \prod_{j=1}^{n} \nu_j{}^{w_j} \right\rangle, \tag{3}$$

$$FFWG(\zeta_1, \zeta_2, ..., \zeta_n) = \overset{n}{\underset{j=1}{\otimes}} w_j \zeta_j = \left\langle \prod_{j=1}^{n} \mu_j{}^{w_j}, \sqrt[3]{1 - \prod_{j=1}^{n} (1 - \nu_j{}^3)^{w_j}} \right\rangle, \tag{4}$$

(w_j being the weightage of ζ_j, $j = 1(1)n$)

Definition 6 [44]: Generalized Dombi (GD) operators introduced by Dombi are described by the formula:

$$GD(y_1, y_2) = \left(1 + \left(\frac{1}{a} \left(\prod_{r=1}^{2} \hbar_a^b(y_r) - 1 \right) \right)^{-\frac{1}{b}} \right)^{-1} \tag{5}$$

$$\text{or} \quad GD(y_1, y_2) = \left(1 + \left(\frac{1}{a} \left(\prod_{r=1}^{2} \lambda\!\!\!^{-}{}_a^b(y_r) - 1 \right) \right)^{\frac{1}{b}} \right)^{-1} \tag{6}$$

where $\hbar_a^b(y_r) = 1 + a\left(\frac{y_r}{1 - y_r}\right)^b$, $\lambda\!\!\!^{-}{}_a^b(y_r) = 1 + a\left(\frac{1 - y_r}{y_r}\right)^b$ $[0 < y_r < 1;\ r = 1, 2]$ with $a > 0$ and $b \geq 1$.

GD operations are noted for their exceptional adaptability in modifying parameters *'a'* and *'b'*, which renders them superior to other existing operations.

2.2 GD operations on FFNs

GD operations on FFNs along with characteristics are presented in this section.

Definition 7: GD operations on FFNs $\zeta_r =< \mu_r, \nu_r > (r=1,2)$ can be proposed as follows:

(i) $$\zeta_1 \oplus \zeta_2 = \left\langle \sqrt[3]{\left(1+\left(\frac{1}{a}\left(\prod_{r=1}^{2} \hbar_a^b(\mu_r^3)-1\right)\right)^{-\frac{1}{b}}\right)^{-1}}, \sqrt[3]{\left(1+\left(\frac{1}{a}\left(\prod_{r=1}^{2} \lambdabar_a^b(\nu_r^3)-1\right)\right)^{\frac{1}{b}}\right)^{-1}} \right\rangle \quad (7)$$

(ii) $$\zeta_1 \otimes \zeta_2 = \left\langle \sqrt[3]{\left(1+\left(\frac{1}{a}\left(\prod_{r=1}^{2} \lambdabar_a^b(\mu_r^3)-1\right)\right)^{\frac{1}{b}}\right)^{-1}}, \sqrt[3]{\left(1+\left(\frac{1}{a}\left(\prod_{r=1}^{2} \hbar_a^b(\nu_r^3)-1\right)\right)^{-\frac{1}{b}}\right)^{-1}} \right\rangle \quad (8)$$

(iii) $$\lambda\zeta_1 = \left\langle \sqrt[3]{\left(1+\left(\frac{1}{a}\left((\hbar_a^b(\mu_1^3))^{\lambda}-1\right)\right)^{-\frac{1}{b}}\right)^{-1}}, \sqrt[3]{\left(1+\left(\frac{1}{a}\left((\lambdabar_a^b(\nu_1^3))^{\lambda}-1\right)\right)^{\frac{1}{b}}\right)^{-1}} \right\rangle (\lambda>0) \quad (9)$$

(iv) $$\zeta_1^{\lambda} = \left\langle \sqrt[3]{\left(1+\left(\frac{1}{a}\left((\lambdabar_a^b(\mu_1^3))^{\lambda}-1\right)\right)^{\frac{1}{b}}\right)^{-1}}, \sqrt[3]{\left(1+\left(\frac{1}{a}\left((\hbar_a^b(\nu_1^3))^{\lambda}-1\right)\right)^{-\frac{1}{b}}\right)^{-1}} \right\rangle (\lambda>0) \quad (10)$$

Theorem 1: Let $\zeta_r =< \mu_r, \nu_r > (r=1,2)$ be two FFNs and $\lambda, \lambda_1, \lambda_2 > 0$. Then,

(i) $\lambda(\zeta_1 \oplus \zeta_2) = (\lambda\zeta_1) \oplus (\lambda\zeta_2)$

(ii) $(\zeta_1 \otimes \zeta_2)^{\lambda} = (\zeta_1^{\lambda}) \otimes (\zeta_2^{\lambda})$

(iii) $(\lambda_1+\lambda_2)\zeta_1 = (\lambda_1\zeta_1) \oplus (\lambda_2\zeta_1)$

(iv) $(\zeta_1 \otimes \zeta_2)^{\lambda_1+\lambda_2} = (\zeta_1^{\lambda_1}) \otimes (\zeta_2^{\lambda_2})$

2.3 FF Generalized-Dombi weighted averaging (FFGDWA) AO

Here, FFGDWA AO is developed, and its characteristics are investigated. Throughout this sub-section, we shall assume that $\zeta_r =< \mu_r, \nu_r > (r=1,2,...,k)$ be a set of FFNs.

Definition 8: If δ_r denotes the weight of ζ_r satisfying $\sum_{r=1}^{k} \delta_r = 1$, then FFGDWA operator is presented as:

$$FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k) = \oplus_{r=1}^{k}(\delta_r \zeta_r) \quad (11)$$

Theorem 2: $FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k)$ is also a FFN and $FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k)$

$$= \left\langle \sqrt[3]{\left(1+\left(\frac{1}{a}\left(\prod_{r=1}^{k} (\hbar_a^b(\mu_r^3))^{\delta_r}-1\right)\right)^{-\frac{1}{b}}\right)^{-1}}, \sqrt[3]{\left(1+\left(\frac{1}{a}\left(\prod_{r=1}^{k} (\lambdabar_a^b(\nu_r^3))^{\delta_r}-1\right)\right)^{\frac{1}{b}}\right)^{-1}} \right\rangle \quad (12)$$

Proof: Follows from Definitions 7 and 8 and Theorem 1.

$FFGDWA$ AO transforms to the (i) FF weighted averaging AO [8] for $a=1, b=1$; (ii) FF Einstein weighted averaging AO [12] for $a=1, b=2$; and (iii) FF Hamachar weighted averaging AO [11] for $b=1$.

Below are the characteristics of FFGDWA operator:

Theorem 3: For an FFN $\zeta_0 =< \mu_0, \nu_0 > (\neq \zeta_r)$, $FFGDWA(\zeta_0 \oplus \zeta_1, \zeta_0 \oplus \zeta_2, ..., \zeta_0 \oplus \zeta_k)$ $= \zeta_0 \oplus FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k)$.

Theorem 4: If $\xi_r = \zeta_0 =< \mu_0, \nu_0 > \forall r$, then $FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k) = \zeta_0$.

Theorem 5: $\zeta^- \prec FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k) \prec \zeta^+$ where $\zeta^- = \left\langle \min_r \mu_r, \max_r \nu_r \right\rangle$ and $\zeta^+ = \left\langle \max_r \mu_r, \min_r \nu_r \right\rangle$.

Theorem 6: If $\zeta'_r =< \mu'_r, \nu'_r > (r = 1, 2, ..., k)$ be another collection of FFNs such that $\mu_r \le \mu'_r, \nu_r \ge \nu'_r \forall r$. Then $FFGDWA(\zeta_1, \zeta_2, ..., \zeta_k) \prec FFGDWA(\zeta'_1, \zeta'_2, ..., \zeta'_k)$.

2.4 FF Generalized-Dombi weighted geometric (FFGDWG) AO

Here, FFGDWG AO is developed, and it's characteristics are surveyed. Throughout this sub-section, we shall assume that $\zeta_r =< \mu_r, \nu_r > (r = 1, 2, ..., k)$ be a set of FFNs.

Definition 9: If δ_r denotes the weight of ζ_r satisfying $\sum_{r=1}^{k} \delta_r = 1$, then FFGDWG operator is presented as:

$$FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k) = \otimes_{r=1}^{k} (\zeta_r^{\delta_r}) \tag{13}$$

Theorem 7: $FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k)$ is also an FFN and $FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k)$

$$= \left\langle \sqrt[3]{\left(1 + \left(\frac{1}{a}\left(\prod_{r=1}^{k} (\hbar_a^b(\mu_r^3))^{\delta_r} - 1\right)\right)^{-\frac{1}{b}}\right)^{-1}}, \sqrt[3]{\left(1 + \left(\frac{1}{a}\left(\prod_{r=1}^{k} (\hbar_a^b(\nu_r^3))^{\delta_r} - 1\right)\right)^{\frac{1}{b}}\right)^{-1}} \right\rangle \tag{14}$$

Proof: Follows from Definitions 7 and 8 and Theorem 1.

$FFGDWG$ AO transforms to the (i) FF weighted geometric AO [8] for $a = 1, b = 1$; (ii) FF Einstein weighted geometric AO [12] for $a = 1, b = 2$; and (iii) FF Hamachar weighted geometric AO [11] for $b = 1$.

The features of the $FFGDWG$ operator are mentioned below.

Theorem 8: For an FFN $\zeta_0 =< \mu_0, \nu_0 > (\ne \zeta_r)$, $FFGDWG(\zeta_0 \oplus \zeta_1, \zeta_0 \oplus \zeta_2, ..., \zeta_0 \oplus \zeta_k) = \zeta_0 \oplus FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k)$.

Theorem 9: If $\xi_r = \zeta_0 =< \mu_0, \nu_0 > \forall r$, then $FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k) = \zeta_0$.

Theorem 10: $\zeta^- \prec FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k) \prec \zeta^+$ where $\zeta^- = \left\langle \min_r \mu_r, \max_r \nu_r \right\rangle$ and $\zeta^+ = \left\langle \max_r \mu_r, \min_r \nu_r \right\rangle$.

Theorem 11: If $\zeta'_r =< \mu'_r, \nu'_r > (r = 1, 2, ..., k)$ be another collection of FFNs such that $\mu_r \le \mu'_r, \nu_r \ge \nu'_r \forall r$, Then $FFGDWG(\zeta_1, \zeta_2, ..., \zeta_k) \prec FFGDWG(\zeta'_1, \zeta'_2, ..., \zeta'_k)$.

2.5 Improved CoCoSo with FF information

Step 1: Formation of the *aggregated FF matrix.*

We consider that A_i $(i = 1, 2, ..., m)$ are alternatives are to be assessed by the specialists/experts E_r (having weight δ_r) $(r = 1, 2, ..., k)$ under FF environment w.r.t the criteria C_j $(j = 1, 2, ..., n)$. The primary assessment results are depicted as the matrices $\Gamma_r = \left[\zeta_r^{(ij)}\right]_{m \times n}$ where $\zeta_r^{(ij)} =< \mu_r^{(ij)}, \nu_r^{(ij)} >$. The aggregated FF decision matrix is $\left[\zeta^{(ij)}\right]_{m \times n} = \left[< \mu^{(ij)}, \nu^{(ij)} >\right]_{m \times n}$, where:

$$\zeta^{(ij)} = FFGDWA(\zeta_1^{(ij)}, \zeta_2^{(ij)}, ..., \zeta_l^{(ij)}) = \oplus_{r=1}^{k} \left(\delta_r \zeta_r^{(ij)}\right), \tag{15}$$

or

$$\zeta^{(ij)} = FFGDWA(\zeta_1^{(ij)}, \zeta_2^{(ij)}, ..., \zeta_l^{(ij)}) = \otimes_{r=1}^{k} \left(\zeta_r^{(ij)}\right)^{\delta_r} \tag{16}$$

Step 2: Obtaining desired degree of *Consensus.*

The measure of correlation $\theta_j^{(r)}$ of the expert E_r under the measure C_j can be expressed as:

$$\theta_j^{(r)} = \frac{\sum_{i=1}^{m}\left[\left(\frac{\vartheta_{ij}^{(r)}}{\vartheta_j^{(r)}} - \frac{1}{m}\sum_{i=1}^{m}\frac{\vartheta_{ij}^{(r)}}{\vartheta_j^{(r)}}\right)\times\left(\frac{\vartheta_{ij}}{\vartheta_j} - \frac{1}{m}\sum_{i=1}^{m}\frac{\vartheta_{ij}}{\vartheta_j}\right)\right]}{\sqrt{\sum_{i=1}^{m}\left(\frac{\vartheta_{ij}^{(r)}}{\vartheta_j^{(r)}} - \frac{1}{m}\sum_{i=1}^{m}\frac{\vartheta_{ij}^{(r)}}{\vartheta_j^{(r)}}\right)^2}\times\sqrt{\sum_{i=1}^{m}\left(\frac{\vartheta_{ij}}{\vartheta_j} - \frac{1}{m}\sum_{i=1}^{m}\frac{\vartheta_{ij}}{\vartheta_j}\right)^2}}\ (j=1,2,...,n;\ r=1,2,...,k) \tag{17}$$

where

$\zeta_r^{(ij)(+)} = \left\langle \max_i \mu_r^{(ij)}, \min_i \nu_r^{(ij)} \right\rangle,\ \zeta_r^{(ij)(-)} = \left\langle \min_i \mu_r^{(ij)}, \max_i \nu_r^{(ij)} \right\rangle,\ \zeta^{(ij)(+)} = \left\langle \max_i \mu^{(ij)}, \min_i \nu^{(ij)} \right\rangle,$

$\zeta^{(ij)(-)} = \left\langle \min_i \mu^{(ij)}, \max_i \nu^{(ij)} \right\rangle,\ \vartheta_{ij}^{(r)} = D(\zeta_r^{(ij)}, \zeta_r^{(ij)(+)}),\ \vartheta_j^{(r)} = D(\zeta_r^{(ij)(+)}, \zeta_r^{(ij)(-)}),$

$\vartheta_{ij} = D(\zeta^{(ij)(+)}, \zeta^{(ij)}),$ and $\vartheta_j = D(\zeta^{(ij)(+)}, \zeta^{(ij)(-)}).$

Next, for the expert E_r, the degree of consensus (DC) $\xi^{(r)}$ is given by:

$$\xi^{(r)} = \frac{1}{n}\sum_{j=1}^{n}\theta_j^{(r)}\ (r=1,2,...,l). \tag{18}$$

If ξ symbolizes the minimum DC, then achieving $\xi^{(r)} \ge \xi$ is necessary. For $\xi^{(r)} < \xi$, the matrices $\left[\zeta_r^{(ij)}\right]_{m\times n}$ must be updated until $\xi^{(r)} \ge \xi$ is satisfied $\forall r$.

Step 3: *Estimation of criteria weights*:

For r^{th} expert, we present the gap between FF information related to A_i and other alternatives under criterion C_j by the subsequent "divergence measure".

$$\varpi_{ij}^r = (m-1)^{-1}\sum_{z=1}^{m} C(\zeta_r^{(ij)}, \zeta_r^{(zj)}) \tag{19}$$

where $C(\zeta_r^{(ij)}, \zeta_r^{(zj)})$ denoting the "cross-entropy measure" is expressed as:

$$C(\zeta_r^{(ij)}, \zeta_r^{(zj)}) = \mu_r^{(ij)}\times\ln\left(\frac{2\mu_r^{(ij)}}{\mu_r^{(ij)}+\mu_r^{(zj)}}\right) + \mu_r^{(zj)}\times\ln\left(\frac{2\mu_r^{(zj)}}{\mu_r^{(ij)}+\mu_r^{(zj)}}\right) + \left(1-\mu_r^{(ij)}\right)\times\ln\left(\frac{1-\mu_r^{(ij)}}{1-\frac{1}{2}\left(\mu_r^{(ij)}+\mu_r^{(zj)}\right)}\right)$$
$$+\left(1-\mu_r^{(zj)}\right)\times\ln\left(\frac{1-\mu_r^{(zj)}}{1-\frac{1}{2}\left(\mu_r^{(ij)}+\mu_r^{(zj)}\right)}\right) + \nu_r^{(ij)}\times\ln\left(\frac{2\nu_r^{(ij)}}{\nu_r^{(ij)}+\nu_r^{(zj)}}\right) + \nu_r^{(zj)}\times\ln\left(\frac{2\nu_r^{(zj)}}{\nu_r^{(ij)}+\nu_r^{(zj)}}\right)$$
$$+\left(1-\nu_r^{(ij)}\right)\times\ln\left(\frac{1-\nu_r^{(ij)}}{1-\frac{1}{2}\left(\nu_r^{(ij)}+\nu_r^{(zj)}\right)}\right) + \left(1-\nu_r^{(zj)}\right)\times\ln\left(\frac{1-\nu_r^{(zj)}}{1-\frac{1}{2}\left(\nu_r^{(ij)}+\nu_r^{(zj)}\right)}\right) \tag{20}$$

The overall divergence generated by the j^{th} criterion is presented by Eqn. (21):

$$\varpi_j^r = \frac{1}{m-1}\sum_{i=1}^{m}\sum_{z=1}^{m} C(\zeta_r^{(ij)}, \zeta_r^{(zj)}) \tag{21}$$

Given the significance of all experts, the total divergences under C_j can be exhibited by Eq. (22):

$$\varpi_j = \sum_{r=1}^{k}\delta_r\varpi_j^r = \sum_{r=1}^{k}\delta_r\frac{1}{m-1}\sum_{i=1}^{m}\sum_{z=1}^{m} C(\zeta_r^{(ij)}, \zeta_r^{(zj)}) \tag{22}$$

For the weighting vector $W = (\varphi_1, \varphi_2, ..., \varphi_n)$, Eq. (23) describes the dispersion measure.

$$Disp(W) = -\sum_{j=1}^{n} \varphi_j \log_2 \varphi_j \tag{23}$$

In order to enhance the capacity for information uptake, both dispersion and total divergence must be maximized. Therefore, using Eq. (24), one can compute the criteria weights.

$$\begin{aligned} &Max\, \chi = \sum_{j=1}^{n} \varphi_j \sum_{r=1}^{k} \delta_r \frac{1}{m-1} \sum_{i=1}^{m} \sum_{z=1}^{m} C(\zeta_r^{(ij)}, \zeta_r^{(zj)}) - \sum_{j=1}^{n} \varphi_j \log_2 \varphi_j \\ &\quad \text{Subject to: } \varphi_j \in \Omega,\ \sum_{j=1}^{n} \varphi_j = 1,\ \varphi_j \geq 0\ \forall j. \end{aligned} \tag{24}$$

where weight of the j^{th} criterion is denoted by φ_j and the collection of all partial weight information of criteria is represented by Ω.

Step 4: *Generate the priority order using FF-improved CoCoSo method:*

Step 4.1: Construct the score matrix (SM) $\left[s(\zeta^{(ij)}) \right]_{m \times n}$ for $\left[\zeta^{(ij)} \right]_{m \times n}$.

Step 4.2: Construct the extended SM (ESM) w.r.t $\left[s(\zeta^{(ij)}) \right]_{m \times n}$.

Here, matrix $\left[s(\zeta^{(ij)}) \right]_{m \times n}$ is upgraded by the inclusion of IDSs and AIDSs, as defined by:

$$\Theta_{+}^{j} = \begin{cases} \max_i s(\zeta^{(ij)}), & \text{if } C_j \in Q_B \\ \min_i s(\zeta^{(ij)}) & \text{if } C_j \in Q_C \end{cases} \tag{25}$$

$$\Theta_{-}^{j} = \begin{cases} \min_i s(\zeta^{(ij)}), & \text{if } C_j \in Q_B \\ \max_i s(\zeta^{(ij)}) & \text{if } C_j \in Q_C \end{cases} \tag{26}$$

Step 4.3: Construct the normalized ESM (NESM) $\left[S(\overline{\zeta}^{(ij)}) \right]_{m \times n}$ corresponding to ESM.

NEM is described as:

$$S(\overline{\zeta}^{(ij)}) = \begin{cases} \dfrac{s(\zeta^{(ij)})}{\max_i s(\zeta^{(ij)})}, & \text{if } C_j \in Q_B \\ \dfrac{\max_i s(\zeta^{(ij)})}{s(\zeta^{(ij)})} & \text{if } C_j \in Q_C \end{cases} \tag{27}$$

Step 4.4: Obtain the weighted NESM $\left[\beta^{(ij)} \right]_{m \times n}$. The elements are depicted as: $\beta^{(ij)} = \varphi_j \times s(\overline{\zeta}^{(ij)})$.

Step 4.5: Calculate UD of each alternative.

UDs of i^{th} alternative w.r.t AID and ID values are formulated as:

$$UD_i^{-} = \frac{1}{\sum_{j=1}^{n} \beta_{-}^{ij}} \sum_{j=1}^{n} \beta^{(ij)}, \quad UD_i^{+} = \frac{1}{\sum_{j=1}^{n} \beta_{+}^{ij}} \sum_{j=1}^{n} \beta^{(ij)} \tag{28}$$

Step 4.6: Compute relative compromise degree (RCD) of each alternative using Eqs. (29)-(31).

$$\alpha_i^{(1)} = \frac{UD_i^{-} + UD_i^{+}}{\sum_{i=1}^{m} \left(UD_i^{-} + UD_i^{+} \right)}, \tag{29}$$

$$\alpha_i^{(2)} = \frac{UD_i^{-}}{\min_i UD_i^{-}} + \frac{UD_i^{+}}{\min_i UD_i^{+}}, \tag{30}$$

$$\alpha_i^{(3)} = \frac{\sigma UD_i^- + (1-\sigma) UD_i^+}{\sigma \max\limits_i UD_i^- + (1-\sigma) \max\limits_i UD_i^+} \tag{31}$$

Here, σ is the preference parameter and $\sigma \in [0,1]$. Generally, $\sigma = 0.5$ is chosen for computations.

Step 4.7: Calculate the "overall compromise degree (OCD)" for each alternative, assessing its importance using Eq. (32).

$$\alpha_i = \sqrt[3]{\alpha_i^{(1)} \alpha_i^{(3)} \alpha_i^{(3)}} + \frac{1}{3}\left(\alpha_i^{(1)} + \alpha_i^{(3)} + \alpha_i^{(3)}\right). \tag{32}$$

Step 4.8: Alternatives are ranked according to their OCD values, with the alternative possessing the highest value identified as the most preferred. A higher OCD value reflects a stronger preference for the respective alternative.

3. Case study

A mid-sized Indian firm, Green Gear Innovations, aims to adopt a multi-criteria framework to evaluate potential outsourcing vendor for precision gearbox manufacturing. The goal is to integrate sustainability metrices into procurement decisions, aligning with global standards. The company prioritizes vendors that align with its sustainability goals while maintaining cost efficiency and quality. The selection process involves for vendor alternatives (*A*1, *A*2, *A*3, *A*4) based on several beneficial criteria namely: carbon emission reduction (*C1*), energy efficiency rating (*C*2), waste recycling rate (*C3*), labor standards compliance (*C4*), health and safety index (*C*5), cost competitiveness (*C6*) and on-time delivery rate (*C7*). To facilitate this process and in order to identify and evaluate the considered vendors, a committee of four experts was formed. The primary evaluations of the four experts are displayed in Tables 1 and 2 respectively.

Table 1
Linguistic variables and their corresponding FFNs

Linguistics variable	FFNs	Linguistics variable	FFN
AI: Absolutely important	<0.95, 0.2>	*F*: Fair	<0.6, 0.7>
VVI: Very-very important	<0.9, 0.3>	*U*: Unimportant	<0.5, 0.8>
VI: Very important	<0.85, 0.4>	*SU*: Slightly unimportant	<0.4, 0.85>
SI: Slightly important	<0.8, 0.5>	*VU*: Very unimportant	<0.3, 0.9
I: Important	<0.75, 0.6>	*VVU*: Very-very unimportant	<0.2, 0.95>

Table 2
Primary assessment outcomes

	Expert 1				Expert 2			
Criteria	A1	A2	A3	A4	A1	A2	A3	A4
C1	I	I	SU	I	I	F	VVI	F
C2	SU	I	SI	I	U	SI	VVI	SI
C3	F	F	I	SU	SU	U	SI	F
C4	F	I	I	F	I	F	SI	SI
C5	I	I	F	I	VI	F	VI	U
C6	I	F	SI	I	F	F	I	SI
C7	F	AI	F	I	VVU	SI	F	AI

Table 2
Continued

	Expert 3				Expert 4			
Criteria	A1	A2	A3	A4	A1	A2	A3	A4
C1	I	VU	I	U	AI	SI	SI	SI
C2	F	U	U	I	I	SI	I	F
C3	I	SI	F	F	SI	F	VU	VU
C4	VI	F	U	U	I	SI	F	I
C5	SI	SI	SI	U	F	F	VI	SI
C6	F	U	SI	AI	U	I	I	SI
C7	U	SI	I	SI	F	I	U	F

4. Result and discussion

4.1 Results

Assuming weights of the specialists as 0.27, 0.30, 0.20, and 0.23 are assumed, the merged FF matrix (Table S1 of the supplementary material) is constructed. Assume the minimum consensus degree is $\xi = 0.50.$ The experts' DCs are calculated (using Eqs. (8) and (9)) as: $\xi^{(1)} = 0.2680,$ $\xi^{(2)} = 0.8098,$ $\xi^{(3)} = 0.5794$ and $\xi^{(4)} = 0.2342$. Since $\xi^{(1)}, \xi^{(4)} < 0.50,$ few observations w.r.t 1st and 4thexperts need to be updated. The updated assessment values are presented in Table S2 of the supplementary material. Next, we form the revised aggregated matrix (Table S3 of the supplementary material). The DCs are computed again using Eqs. (17) and (18) as: $\xi^{(1)} = 0.6479,$ $\xi^{(2)} = 0.7154,$ $\xi^{(3)} = 0.5371$ and $\xi^{(4)} = 0.5473$. Since $\xi^{(r)} > 0.50$ $(r = 1, 2, 3, 4),$ desired consensus is achieved.

Next, considering that the partial weight information of criteria is provided by: $\Omega = \{\varphi_1 > \varphi_3,\ \varphi_5 > \varphi_6,\ \varphi_4 > \varphi_7\}$ Then based on Eqs. (19)-(24), we obtain:

$$
\begin{aligned}
Max\ Z = \ & \varphi_1 \times \left(0.6794 - log_2\varphi_1\right) + \varphi_2 \times \left(0.3839 - log_2\varphi_2\right) + \varphi_3 \times \left(0.\ 4920 - log_2\varphi_3\right) \\
& + \varphi_4 \times \left(0.2862 - log_2\varphi_4\right) + \varphi_5 \times \left(0.9196 - log_2\varphi_5\right) + \varphi_6 \times \left(0.3726 - log_2\varphi_6\right) \\
& + \varphi_7 \times \left(1.4134 - log_2\varphi_7\right)
\end{aligned}
$$

$$
\text{Subject to} \quad \begin{cases} \varphi_1 > \varphi_3,\ \varphi_5 > \varphi_6,\ \varphi_4 > \varphi_7, \\ \varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 + \varphi_5 + \varphi_6 + \varphi_7 = 1, \\ \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, \varphi_7 \geq 0 \end{cases}
$$

Solving, we get: $Max\ \chi$ =3.6221 and $\varphi_1 = 0.3160,$ $\varphi_2 = 0.1109,$ $\varphi_3 = 0.1093,$ $\varphi_4 = 0.1254,$ $\varphi_5 = 0.1065,$ $\varphi_6 = 0.1064, \varphi_7 = 0.1254$.

Next, SM corresponding to the aggregated FF decision matrix is constructed, and the extended SM (ESM) is formed based on Eqs. (25) and (26). Eq. (27) is applied to construct normalized ESM (NESM) and weighted NESM. UD of each alternative is then obtained using Eq. (28), and subsequently, RCD of each alternative is calculated using Eqs. (29), (30), & (31). Finally, OCD of each alternative is computed by making use of Eq. (32). UDs, RCDs, and OCDs of the alternatives are presented in Table 3. This table also shows the priority order, and the most favourable MOV is "A4".

Table 3
Results of FF-CoCoSo model

Alternative	UDs		RCD			OCD
	UD_i^-	UD_i^+	$\alpha_i^{(1)}$	$\alpha_i^{(2)}$	$\alpha_i^{(3)}$	
A1	1.33848	0.76336	0.24529	2.04570	0.94854	1.86051
A2	1.30858	0.74631	0.23981	2.00000	0.92735	1.81895
A3	1.39849	0.79759	0.25629	2.13741	0.99107	1.94393
A4	1.41110	0.80478	0.25860	2.15668	1.00000	1.96145

4.2 Sensitivity analysis (SA)

SA is undertaken to investigate the impact of criteria weights on alternative prioritizations. In this process, assumptions are made regarding the incremental increase and decrease of each criterion weight by 10%, 20%, 30%, 40%, and 50%, respectively. Results are shown in Fig. 1 to Fig. 7. In each figure, ten scenarios of variations (S1: 50% decrease, S2: 40% decrease, S3: 30% decrease, S4: 20% decrease, S5: 10% decrease, S6: 10% increase, S7: 20% increase, S8: 30% increase, S9: 40% increase, and S10: 50% increase) along with the original value of a particular criteria weight have been presented. "Spearman's rank correlation coefficient" (SRCC) values [45] corresponding to the scenarios are presented in Tables S4-S10 (supplementary material). The average SRCC values for weight variations of C1, C2, C3, C4, C5, C6, and C7 are 0.76, 0.98, 0.92, 1.00, 0.92, 0.98, and 0.80, respectively. A mean SRCC exceeding 0.75 indicates a "strong correlation" [45] among the alternatives' order of importance. Additionally, out of 70 scenarios (accounting for all criteria), it is evident that (i) A4 is ranked first in 53 scenarios (75% of scenarios), and (ii) the ranking orders match those obtained in Sub-section 6.1 for 44 scenarios (62% of scenarios). These findings collectively advocate the credibility of the obtained results.

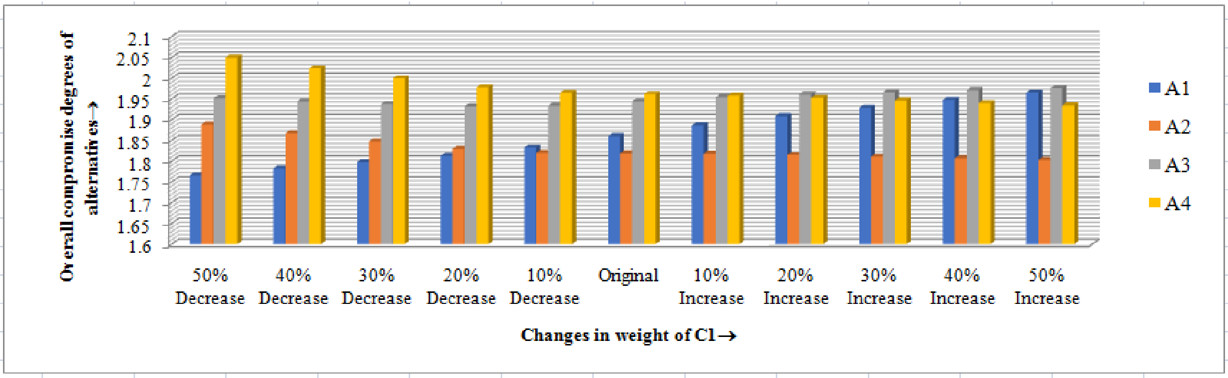

Fig. 1. Sensitivity analysis of C1

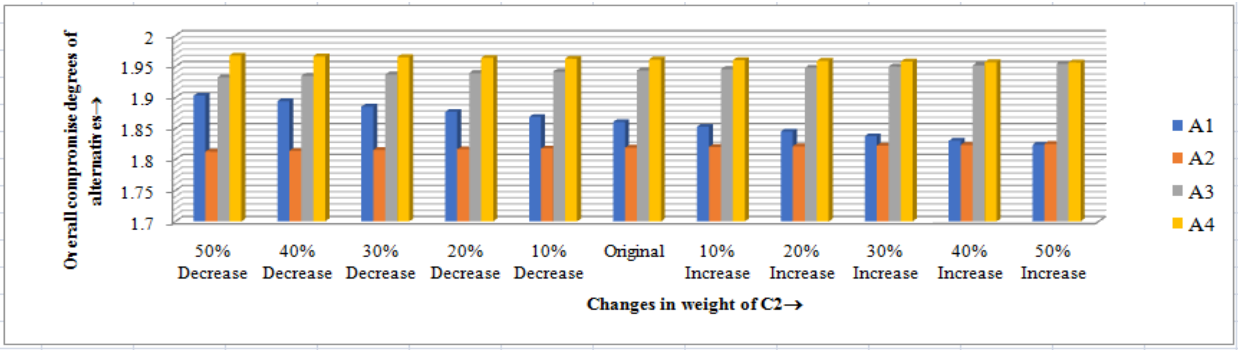

Fig. 2. Sensitivity analysis of C2

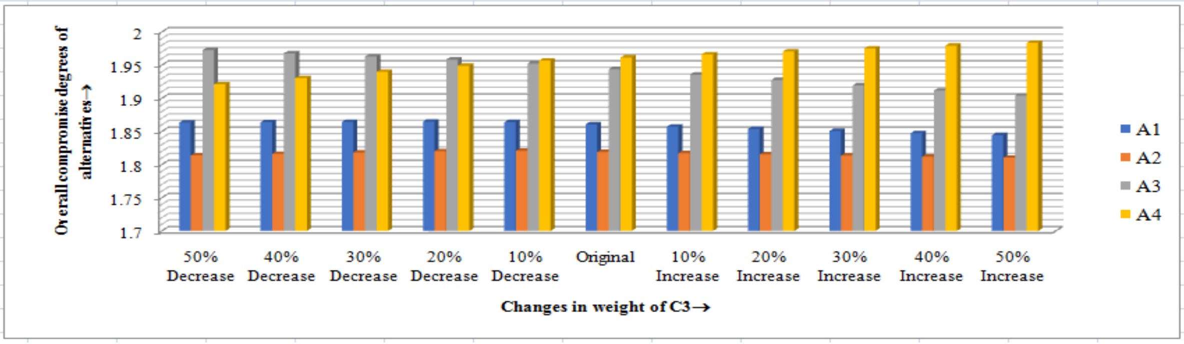

Fig. 3. Sensitivity analysis of C3

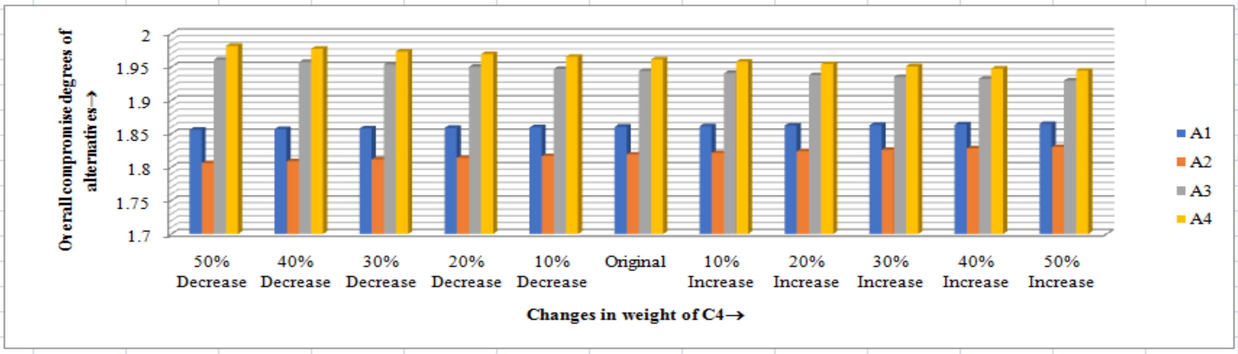

Fig. 4. Sensitivity analysis of C4

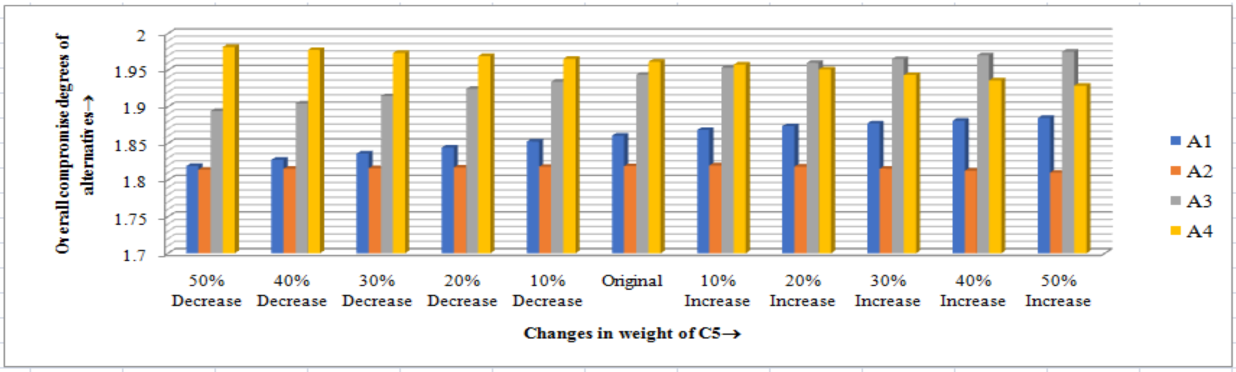

Fig. 5. Sensitivity analysis of C5

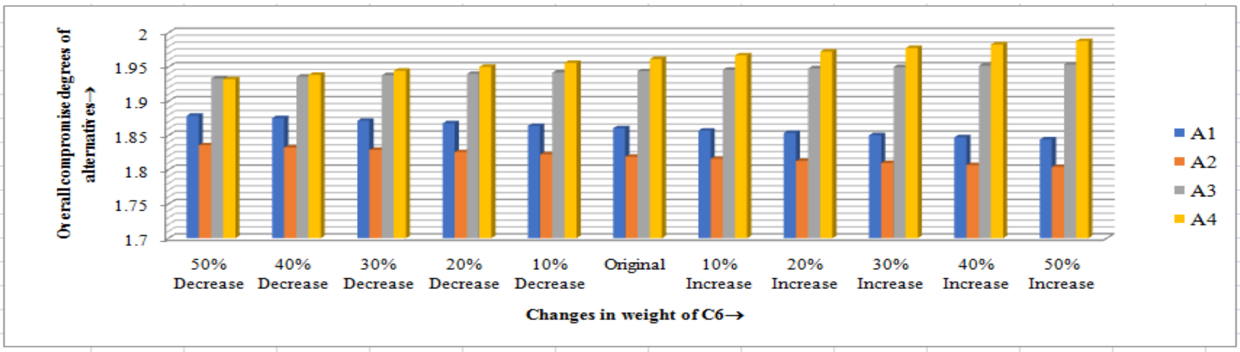

Fig. 6. Sensitivity analysis of C6

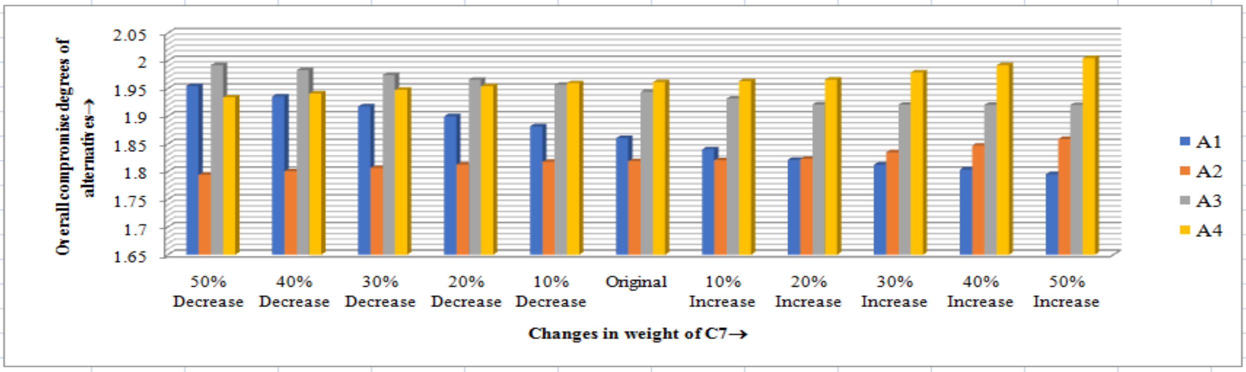

Fig. 7. Sensitivity analysis of C7

4.3 Comparative study

Comparing the proposed FF decision support model with other existing FF models like FF-MARCOS, FF-TOPSIS, and FF-COPRAS is the goal of this section. Tables S11-S13 (supplementary material) illustrate the specific outcomes of the application of these techniques to the case study covered in Section 5. The rating rankings provided by FF-MARCOS, FF-TOPSIS, and FF-COPRAS models are as follows: $A_4 \succ A_3 \succ A_2 \succ A_1, A_3 \succ A_4 \succ A_1 \succ A_2, A_4 \succ A_1 \succ A_3 \succ A_2$. However, the proposed model has generated a ranking order of $A_4 \succ A_3 \succ A_1 \succ A_2$ which differs slightly from these orders. Nonetheless, the SRCC value of each of FF-MARCOS, FF-TOPSIS, and FF-COPRAS in comparison to the proposed method is 0.80 which indicates a strong correlation between the proposed and existing methods. The primary benefits of the proposed method are as follows:

i. The Generalized-Dombi AOs, the proposed model offers a significant degree of flexibility for experts to select the most suitable parameters according to the consensus-reaching process.
ii. The proposed model incorporates decision-makers' processes for reaching consensus, a feature absents in existing FFS-based. Biases are mitigated by the proposed model, making the process more rational and feasible.
iii. The proposed model accounts for the challenges in group decision-making scenarios, computes uncertain information, and emphasizes on appropriate estimation of criteria weights.
iv. The proposed FF-improved CoCoSo model is free from the shortcoming occurred in CoCoSo as it includes both IDSs and AIDSs and UD of each alternative related to IDSs and AIDSs.

5. Conclusions

The process of choosing an appropriate MOV is intricate and multidimensional. This problem is brought on by the engagement of multiple specialists, a large number of possible vendors, and multiple criteria that must be taken into account at the same time. The decision-making process is further complicated by the possibility that various experts would consider criteria differently. Because expert assessments are frequently subjective and provide ratings that are imprecise and occasionally ambiguous, the process becomes even more unpredictable. The use of MCDM methods in the manufacturing environment has become more and more important as a result of these difficulties. Even in cases when expert opinions are ambiguous or contradictory, MCDM methods are made to manage such complications by methodically assessing options according to a number of criteria.

In considering the systematic expression of uncertain and imprecise information, FFS is viewed as an advanced iteration of FS and intuitionistic FS. Existing AOs developed for amalgamating FF information have shown limited flexibility and generality. Building upon the concept of GD operations, new operations between FF numbers have been introduced in this study. Subsequently, FF Generalized-Dombi weighted averaging AO has been formulated, and its fundamental characteristics have been discussed. Then the proposed FF-improved CoCoSo model is applied for evaluating MOVs to improve the sustainability, and competitiveness of a manufacturing firm. To improve the manufacturing organization's overall performance and efficiency, the case study looks at the important management factors that can be utilized to implement the best MOV. This consensus-based process reduces the effects of biased experts. The estimation of criteria weights is achieved through an optimization model that incorporates cross-entropy and dispersion measures, while the effectiveness of the developed model in addressing group decision-making problems in FF scenarios is demonstrated by comparative analyses.

The future research can explore extending the model to dynamic decision-making environments, where criteria weights or vendor performance fluctuate over time in response to SC volatility. Additionally, incorporating advanced FSs such as q-rung orthopair or spherical may enhance its ability to handle uncertainty. Introducing hierarchical criteria structures could also improve its effectiveness in evaluating complex manufacturing scenarios.

Author Contributions

Conceptualization, A. S. and P.C.; methodology, A.S.; software, A.S.; validation, A.S.; formal analysis, A.S. and P.C.; resources, A.S. and P.C.; data curation, A.S.; writing—original draft preparation, A.S. and P.C.; writing—review and editing, P.C.; visualization, A.S. and P.C. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement

This study did not report any experimental data.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Lyutak, O., Baula, O., & Tatarchuk, D. (2024). Outsourcing in the international business environment: Prospects and challenges. Economic Space, 191, 275-279. https://doi.org/10.32782/2224-6282/191-45

[2] Wu, D., Tannen, M. B., Ndumbe, N. A., Ivanov, S. V., & Xu, F. (2023). Contract manufacturing, market competition, and labor productivity in US manufacturing industries. Operations Management Research, 16, 377-390. https://doi.org/10.1007/s12063-022-00317-0

[3] Malviya, R. K., Kant, R., & Gupta, A. D. (2018). Evaluation and selection of sustainable strategy for green supply chain management implementation. Business Strategy and the Environment, 27, 475-502. https://doi.org/10.1002/bse.2016
[4] Bhattacharya, A., & Singh, P. J. (2019). Antecedents of agency problems in service outsourcing. International Journal of Production Research, 57, 4194-4210. https://doi.org/10.1080/00207543.2018.1506179
[5] Lin, Y. T., Lin, C. L., Yu, H. C., & Tzeng, G. H. (2010). A novel hybrid MCDM approach for outsourcing vendor selection: A case study for a semi-conductor company in Taiwan. Expert Systems with Applications, 37, 4796-4804. https://doi.org/10.1016/j.eswa.2009.12.036
[6] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X
[7] Yager, R. R. (2014). Pythagorean membership grades in multi-criteria decision-making. IEEE Transactions on Fuzzy Systems, 22, 958-965. https://doi.org/10.1109/TFUZZ.2013.2278989
[8] Senapati, T., & Yager, R. R. (2020). Fermatean fuzzy sets. Journal of Ambient Intelligence and Humanized Computing, 11, 663-674. https://doi.org/10.1007/s12652-019-01377-0
[9] Senapati, T., & Yager, R. R. (2019). Fermatean fuzzy weighted averaging/geometric operators and its application in multi-criteria decision-making methods. Engineering Applications of Artificial Intelligence, 85, 112-121. https://doi.org/10.1016/j.engappai.2019.05.012
[10] Aydemir, S. B., & Gunduz, S. Y. (2020). Fermatean fuzzy TOPSIS method with Dombi aggregation operators and its application in multi-criteria decision making. Journal of Intelligent and Fuzzy Systems, 39(1), 851-869. https://doi.org/10.3233/JIFS-191763
[11] Hadi, A., Khan, W., & Khan, A. (2021). A novel approach to MADM problems using Fermatean fuzzy Hamacher aggregation operators. International Journal of Intelligent Systems, 36(7), 3464-3499. https://doi.org/10.1002/int.22423
[12] Rani, P., & Mishra, A. R. (2021). Fermatean fuzzy Einstein aggregation operators based MULTIMOORA method for electric vehicle charging station selection. Expert Systems with Applications, 182, 115267. https://doi.org/10.1016/j.eswa.2021.115267
[13] Gul, S. (2021). Fermatean fuzzy set extensions of SAW, ARAS, and VIKOR with applications in COVID-19 testing laboratory selection problem. Expert Systems, 38(8), e12769. https://doi.org/10.1111/exsy.12769
[14] Sari, I. U., & Sargin, S. N. (2021). Prioritization of R & D project using Fermatean fuzzy MARCOS method. In C. Kahraman, S. Cebi, S. Cevik Onar, B. Oztaysi, A. C. Tolga, & I. U. Sari (Eds.), Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation (pp. 569-577). Springer, Cham. https://doi.org/10.1007/978-3-030-85577-2_67
[15] Kamali Saraji, M., Streimikiene, D., & Kyriakopoulos, G. L. (2021). Fermatean Fuzzy CRITIC-COPRAS Method for Evaluating the Challenges to Industry 4.0 Adoption for a Sustainable Digital Transformation. Sustainability, 13(17), 9577. https://doi.org/10.3390/su13179577
[16] Mishra, A. R., & Rani, P. (2021). Multi-criteria healthcare waste disposal location selection based on Fermatean fuzzy WASPAS method. Complex and Intelligent Systems, 7, 2469-2484. https://doi.org/10.1007/s40747-021-00407-9
[17] Gul, M., Lo, H. W., & Yucesan, M. (2021). Fermatean fuzzy TOPSIS-based approach for occupational risk assessment in manufacturing. Complex and Intelligent Systems, 7, 2635-2653. https://doi.org/10.1007/s40747-021-00417-7
[18] Mishra, A. R., Rani, P., & Pandey, K. (2022). Fermatean fuzzy CRITIC-EDAS approach for the selection of sustainable third-party reverse logistics providers using an improved generalized score function. Journal of Ambient Intelligence and Humanized Computing, 13, 295-311. https://doi.org/10.1007/s12652-021-02902-w
[19] Mishra, A. R., Rani, P., Saha, A., Senapati, T., Hezam, I. M., & Yager, R. R. (2022). Fermatean fuzzy copula aggregation operators and similarity measures based complex proportional assessment approach for renewable energy source selection. Complex and Intelligent Systems. https://doi.org/10.1007/s40747-022-00743-4
[20] Sari, I. U., Kuchta, D., & Sergi, D. (2022). Analysis of intelligent software implementations in air cargo using Fermatean fuzzy CODAS method. In C. Kahraman & S. Aydın (Eds.), Intelligent and Fuzzy Techniques in Aviation 4.0. Studies in Systems, Decision and Control (pp. 147-173). Springer, Cham. https://doi.org/10.1007/978-3-030-75067-1_7
[21] Aytekin, A., Gorcun, O. F., Ecer, F., Pamucar, D., & Karamasa, C. (2023). Evaluation of the pharmaceutical distribution and warehousing companies through an integrated FF entropy-WASPAS approach. Kybernetes, 52(11), 5561-5592. http://dx.doi.org/10.1108/K-04-2022-0508
[22] Akthar, M., & Asim, M. (2024). Fuzzy causal mode of enterprise flexibility dimensions using Fermatean Fuzzy DEMATEL: A case study of Indian pharmaceutical Industry. Benchmarking: An International Journal. https://doi.org/10.1108/BIJ-07-2023-0462

[23] Hezam, I. M., Basua, D., Mishra, A. R., Rani, P., & Cavallaro, F. (2023a). Intuitionistic fuzzy gained and lost dominance score based on symmetric point criterion to prioritize zero carbon measures for sustainable urban transportation. Kybernetes. https://doi.org/10.1108/K-03-2023-0380
[24] Hezam, I. M., Mishra, A. K., Pamucar, D., Rani, P., & Mishra, A. R. (2023). Standard deviation and rank sum-based MARCOS model under intuitionistic fuzzy information for hospital site selection. Kybernetes. https://doi.org/10.1108/K-01-2023-0136
[25] Yazdani, M., Zarate, P., Zavadskas, E. K., & Turskis, Z. (2019). A combined compromise solution (CoCoSo) method for multi-criteria decision-making problems. Management Decision, 57(9), 2501-2519. http://dx.doi.org/10.1108/MD-05-2017-0458
[26] Wen, Z., Liao, H., Zavadskas, E. K., & Al-Barakati, A. (2019). Selection third-party logistics service providers in supply chain finance by a hesitant fuzzy linguistic combined compromise solution method. *Economic Research-Ekonomska Istraživanja, 32*(1), 4033-4058. https://doi.org/10.1080/1331677X.2019.1678502
[27] Peng, X., Zhang, X., & Luo, Z. (2020). Pythagorean fuzzy MCDM method based on CoCoSo and CRITIC with score function for 5G industry evaluation. Artificial Intelligence Review, 53(5), 3813-3847. https://doi.org/10.1007/s10462-019-09780-x
[28] Yazdani, M., Wen, Z., Liao, H., Banaitis, A., & Turskis, Z. (2019). A grey combined compromise solution (CoCoSo-G) method for supplier selection in construction management. Journal of Civil Engineering and Management, 25(8), 858-874. https://doi.org/10.3846/jcem.2019.11309
[29] Peng, X., & Huang, H. (2020). Fuzzy decision-making method based on CoCoSo with critic for financial risk evaluation. Technological and Economic Development of Economy, 26(4), 695-724. https://doi.org/10.3846/tede.2020.11920
[30] Rani, P., & Mishra, A. R. (2020). Novel single-valued neutrosophic combined compromise solution approach for sustainable waste electrical and electronics equipment recycling partner selection. IEEE Transactions on Engineering Management, 69(6), 3139-3153. http://dx.doi.org/10.1109/TEM.2020.3033121
[31] Liu, P., Rani, P., & Mishra, A. R. (2021). A novel Pythagorean fuzzy combined compromise solution framework for the assessment of medical waste treatment technology. Journal of Cleaner Production, 292, 126047. http://dx.doi.org/10.1016/j.jclepro.2021.126047
[32] Mishra, A. R., Rani, P., Saha, A., Hezam, I. M., Pamucar, D., & Pandey, K. (2022c). Assessing the adaption of Internet of Things (IoT) barriers for smart cities waste management using Fermatean fuzzy combined compromise solution approach. IEEE Access, 10, 37109-37130. http://dx.doi.org/10.1109/ACCESS.2022.3164096
[33] Narang, M., Joshi, M. C., Bisht, K., & Pal, A. (2022). Stock portfolio selection using a new decision-making approach based on the integration of fuzzy CoCoSo with Heronian mean operator. Decision Making: Applications in Management and Engineering, 5(1), 90-112. https://doi.org/10.31181/dmame0310022022n
[34] Lai, H., Liao, H., Long, Y., & Zavadskas, E. K. (2022). A hesitant Fermatean fuzzy CoCoSo method for group decision-making and an application to blockchain platform evaluation. International Journal of Fuzzy Systems, 24, 2643-2661. http://dx.doi.org/10.1007/s40815-022-01319-7
[35] Wang, H., Mahmood, T., & Ullah, K. (2023). Improved CoCoSo Method Based on Frank Soft-max Aggregation Operators for T-Spherical Fuzzy Multiple Attribute Group Decision-Making. International Journal of Fuzzy Systems, 25, 1275-1310. https://doi.org/10.1007/s40815-022-01442-5
[36] Tripathi, D. K., Nigam, S. K., Rani, P., & Shah, A. R. (2023). New Intuitionistic fuzzy parametric divergence measures and score function based CoCoSo method for decision-making problems. Decision Making: Applications in Management and Engineering, 6(1), 535-563. https://doi.org/10.31181/dmame0318102022t
[37] Pamučar, D., Gökaşar, İ., Torkayesh, A. E., Deveci, M., Martínez, L., & Wu, Q. (2023). Prioritization of unmanned aerial vehicles in transportation systems using the integrated stratified fuzzy rough decision-making approach with the Hamacher operator. Information Sciences, 622, 374-404. http://dx.doi.org/10.1016/j.ins.2022.11.143
[38] Alnoor, A., Muhsen, Y. R., Husin, N. A., Chew, X., Zolkepli, M. B., & Manshor, N. (2024). Z-Cloud Rough Fuzzy-Based PIPRECIA and COCOSO integration to assess agriculture decision support tools. International Journal of Fuzzy Systems. https://doi.org/10.1007/s40815-024-01771-7
[39] Liu, D., & Huang, A. (2020). Consensus reaching process for fuzzy behavioural TOPSIS method with probabilistic linguistic q-rung orthopair fuzzy sets based on correlation measure. International Journal of Intelligent Systems, 35(3), 494-528. https://doi.org/10.1002/int.22215
[40] Saha, A., Debnath, B. K., Chatterjee, P., Panaiyappan, A. K., Das, S., & Anusa, G. (2024). Generalized Dombi based probabilistic hesitant fuzzy consensus reaching model for supplier selection under healthcare supply chain framework. Engineering Applications of Artificial Intelligence, 133, 107966. http://dx.doi.org/10.1016/j.engappai.2024.107966
[41] Saha, A., Gorcun, O. F., Pamucar, D., Arya, L., & Simic, V. (2024). Evaluation of shared micro-mobility systems for sustainable cities by using a consensus based Fermatean fuzzy multiple objective optimization and full multiplicative

form. Engineering Applications of Artificial Intelligence, 134, 108662. http://dx.doi.org/10.1016/j.engappai.2024.108662

[42] Görçün, O. F., Saha, A., & Ecer, F. (2025). Evaluation of crawler cranes for large-scale Construction and infrastructure projects: An Intuitionistic fuzzy consensus-based approach. Journal of Industrial Information Integration, 44, 100784. https://doi.org/10.1016/j.jii.2025.100784

[43] Ecer, F., Saha, A., Dinçer, H., & Yüksel, S. (2025). Evaluating energy management investments for sustainable transition of smart cities through a new Intuitionistic fuzzy consensus-based CoCoSo methodology. Renewable Energy, 244, 122655. https://doi.org/10.1016/j.renene.2025.122655

[44] Dombi, J. (2009). The generalized Dombi operator family and the multiplicative utility function. Studies in Computational Intelligence. https://doi.org/10.1007/978-3-642-00448-3-6

[45] Saha, A., Simic, V., Senapati, T., Dabic-Miletic, S., & Ala, A. (2022). A dual hesitant fuzzy sets-based methodology for advantage prioritization of zero-mile delivery solutions for sustainable city logistics. IEEE Transactions on Fuzzy Systems, 31(2), 7-20. http://dx.doi.org/10.1109/TFUZZ.2022.3164053