SCIENTIFIC OASIS

Spectrum of Mechanical Engineering and Operational Research

Journal homepage: www.smeor-journal.org
ISSN: 3042-0288

SITW method: a new approach to re-identifying multi-criteria weights in complex decision analysis

Bartłomiej Kizielewicz[1], Wojciech Sałabun[1,*]

[1] National Institute of Telecommunications, Szachowa 1, Warsaw, 04-894, Poland

ARTICLE INFO

Article history:
Received 15 April 2024
Received in revised form22 August 2024
Accepted 30 August 2024
Available online 1 September 2024

Keywords:
Criteria weights; TOPSIS; MCDA; MCDM; SITW

ABSTRACT

Multi-Criteria Decision Analysis (MCDA) addresses complex decision-making problems across various fields such as logistics, management, medicine, and sustainability. MCDA tools provide a structured approach to evaluating decisions with multiple conflicting criteria, assisting decision-makers in navigating intricate scenarios. Engaging experts is crucial for identifying multi-criteria models due to the diverse aspects of decision-making problems. Techniques such as pairwise comparisons and criterion weight assignment are commonly used to incorporate expert knowledge into decision models. Criterion weight assignment allows experts to indicate the importance of each criterion; however, issues can arise if model parameters are lost or experts become unavailable. To mitigate these issues, techniques like entropy or standard deviation can determine weights without direct expert input. In this context, the Stochastic Identification of Weights (SITW) method utilizes existing assessment samples to re-identify models and obtain weights that replicate the rankings of a reference model. This study compares information-based methods (Entropy, STD) with the SITW method in re-identifying the TRI medical function as a benchmark. The effectiveness of these methods is evaluated using Spearman's weighted correlation coefficient across various scenarios and alternative numbers. Results indicate that the SITW method provides more significant results than other methods in identifying multi-criteria weights by leveraging previously evaluated alternatives. Future research could explore broader approaches and uncertainty scenarios to ensure comprehensive decision support in complex contexts.

1. Introduction

Multi-Criteria Decision Analysis (MCDA) is a research area that focuses on the practical solution of complex decision-making problems, which is common in diverse fields such as logistics [1, 2], manage-

*Wojciech Sałabun.
E-mail address: w.salabun@il-pib.pl

https://doi.org/10.31181/smeor11202419

ment [3, 4], medicine [5, 6], and sustainability [7, 8]. MCDA tools are used to support the decision-making process in situations where there are multiple conflicting criteria, challenging traditional analytical methods. MCDA tools offer a structured and systematic way to analyze decisions, considering the diversity of criteria and decision makers' preferences.

In multi-criteria decision analysis issues, complex decision-making problems often require the involvement of experts. Their participation is essential in identifying multi-criteria models due to the diversity of decision-making aspects [5, 9]. Various methods of expressing their knowledge are used to extrude the expert's knowledge into the decision model, which can be applied depending on the context of the problem. One popular approach is pairwise comparisons of reference points, which allow relative evaluation of alternative features [10].

However, criterion weights are the most classic method of transferring expert knowledge in the context of multi-criteria decision analysis [11, 12]. This technique involves assigning each criterion a weight that reflects its importance in the context of the decision to be made. Experts can determine these weights, assess their experience, and analyze the data.

Once an expert has developed an identified model, potential problems may be associated with loss of model parameters or lack of expert availability. This is especially true in long-term projects or situations where the expert may be unavailable for various reasons, such as time constraints, employment change, or other commitments. In order to address the problem of lack of expert knowledge [13], techniques are being used to determine weights based on measures of information that allow criteria weights to be determined objectively without involving an expert in each stage of the analysis. One such technique is the determination of criterion weights using entropy or standard deviation [14].

However, methods based on measures of information mainly focus on analyzing input data. In some cases, it may be beneficial to use assessment samples that are already available to optimize the model. This raises the possibility of Stochastic Identification of Weights (SITW) [15], an innovative approach that uses the underlying mechanism of the selected MCDA technique and optimization techniques to select criteria weights to achieve rankings as close as possible to samples that have already been evaluated.

Therefore, this study compares information-based methods, such as Entropy and STD, with the Stochastic Identification of Weight (SITW) method in re-identifying TRI medical functions. The study aimed to evaluate the effectiveness of these methods under different scenarios, considering different numbers of alternatives. The medical function of TRI was adopted as a benchmark, allowing for a comparison of results and identifying the best strategies. The study was conducted for different numbers of alternatives to understand in which cases the methods are more beneficial and effective. To this end, Spearman's weighted correlation coefficient (r_w) was used to compare the similarity of the rankings generated by each method.

The rest of the article is organized as follows. Section 2 presents preliminaries related to the methods that were used in the study. Section 3 presents the research, the results obtained from it, and analyses of the results. Section 4 presents the conclusions.

2. Preliminaries

2.1 Technique for Order Preference by Similarity to an Ideal Solution

The Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) functions within a structured framework revolving around reference points, employing a logical approach to assess alternatives [16, 17]. At the core of this method lie two critical reference points: the Positive Ideal Solution (PIS) and the Negative Ideal Solution (NIS). TOPSIS executes the evaluation process by gauging the proximity of alternatives to these reference points. The entire procedure of TOPSIS can be delineated

as follows, encompassing several key steps:
Step 1. Normalize the decision matrix by using min-max normalization. The values of benefit type criteria are normalized using the Eq. (1), while the values of cost type criteria are normalized using the Eq. (2).

$$r_{ij} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)} \tag{1}$$

$$r_{ij} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)} \tag{2}$$

Step 2. Calculation of a weighted normalized decision matrix by Eq. (3):

$$v_{ij} = w_i \cdot r_{ij}, \quad i = 1, \ldots, n \quad j = 1, \ldots, J \tag{3}$$

Step 3. Identification of the Positive and Negative Ideal Solutions for a defined decision-making problem with Eq. (4):

$$\begin{aligned} v_j^+ &= \{v_1^+, v_2^+, \ldots, v_n^+\} = \{\max_j(v_{ij})\} \\ v_j^- &= \{v_1^-, v_2^-, \ldots, v_n^-\} = \{\min_j(v_{ij})\} \end{aligned} \tag{4}$$

where I^P stands for profit type criteria and I^C for cost type.

Step 4. Calculation of the Positive and Negative Distances using the n-dimensional Euclidean distance with Eq. (5):

$$\begin{aligned} D_j^* &= \sqrt{\textstyle\sum_{i=1}^n (v_{ij} - v_i^*)^2}, \quad j = 1, \ldots, J \\ D_j^- &= \sqrt{\textstyle\sum_{i=1}^n \left(v_{ij} - v_i^-\right)^2}, \quad j = 1, \ldots, J \end{aligned} \tag{5}$$

Step 5. Calculation of the relative closeness to the Ideal Solution by Eq. (6):

$$C_j^* = \frac{D_j^-}{\left(D_j^* + D_j^-\right)}, \quad j = 1, \ldots, J \tag{6}$$

2.2 *Entropy weights*

The entropy weighting approach derives from Shannon's information uncertainty measure, offering benefits such as risk reduction and efficiency enhancement [14]. The methodology of entropy weights can be outlined through the following steps:

Step 1. Normalization of the decision matrix $X = x_{ij}; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$, where m is the number of alternatives, n is the number of criteria. This normalization can be represented by the Eq. (7).

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad i = 1, \ldots, m; j = 1, \ldots, n \tag{7}$$

Step 2. Calculation of the entropy measure E_j for each criterion according to the Eq. (8).

$$E_j = -\frac{\sum_{i=1}^{m} p_{ij} \ln (p_{ij})}{\ln(m)} \quad j = 1, \ldots, n \tag{8}$$

Step 3. Derive weights based on the calculated entropy for each criterion by Eq. (9).

$$w_j = \frac{1 - E_j}{\sum_{i=1}^{n} (1 - E_i)} \quad j = 1, \ldots, n \tag{9}$$

2.3 *Standard deviation weights*

The standard deviation weighting method is based on the statistical measure of standard deviation. It assigns small weights to a criterion that has similar values across variants [14]. This method can be presented in two steps:

Step 1. Calculate the standard deviation measure for all criteria according to the Eq. (10).

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{m} (x_{ij} - \overline{x_j})^2}{m}} \quad j = 1, \ldots, n \tag{10}$$

where x_{ij} is the value from the decision matrix for $i-th$ alternative and $j-th$ criterion.

Step 2. Derive the weights based on the values of the standard deviation measure with Eq. (11).

$$w_j = \frac{\sigma_j}{\sum_{j=1}^{n} \sigma_j} \quad j = 1, \ldots, n \tag{11}$$

2.4 *Stochastic IdenTification of Weights*

This section will introduce a method for re-identifying weights called Stochastic Identification of Weights (SITW), employing a stochastic optimization technique known as Particle Swarm Optimization (PSO). The SITW method was initially proposed in the study by [15] as a solution to the challenge of limited input data availability concerning weights, which are pivotal for many MCDA methodologies. By leveraging stochastic methods tailored for optimization tasks, it becomes feasible to determine optimal weights facilitating the subsequent re-identification of MCDA models. Within the scope of this study, which exclusively concentrates on the PSO approach, the SITW method can be delineated through the following steps:

Step 1. Select a dataset. The dataset should contain criteria vectors (C), a criteria types vector (T), and a ranking vector (R).

Step 2. Select a stochastic optimization method. In this step, choose a stochastic method for solving the optimization problem and select its parameters. In this paper, Particle swarm optimization (PSO) was selected as the stochastic optimization method, whose algorithm can be presented as follows:

Algorithm 1 Particle Swarm Optimization (PSO)

Input: $f, N, D, [a, b], max\_iterations, w, c_1, c_2$
Initialize $X, V, P, P_{\text{value}}, G, G_{\text{value}}$
for $iteration \leftarrow 1$ to $max\_iterations$ **do**
  for each particle i **do**
    Update velocity and position
    Clip position to within bounds
    Evaluate objective function
    if $f_i > P_{\text{value}}[i]$ **then**
      Update personal best
    end if
  end for
  Update global best
  if convergence_criteria_met() **then**
    break
  end if
end for
Output: G, G_{value}

Step 3. Model training. Training the model is done using the stochastic optimization algorithm and the fitness function, which can be represented as follows:

Algorithm 2 Fitness Function

procedure Fitness($solutions$):
  $solutions \leftarrow solutions/\text{sum}(solutions)$
  $preference \leftarrow \text{base}(C, solutions, T)$
  return rw(base.rank($preference$), R)
end procedure

2.5 *Weighted Spearman's correlation coefficient*

The Weighted Spearman's correlation coefficient (r_W), introduced by [18], builds upon the conventional Spearman coefficient by incorporating weights. These weights strategically amplify the influence of ranking changes at the forefront, thereby impacting the final correlation value. This correlation computation involves two rankings, each of size N, where x_i represents the position in the first ranking and y_i denotes the position in the second ranking (12). The resulting r_W correlation coefficient ranges from -1 to 1. A value of one indicates identical rankings, minus one suggests reversed rankings, and zero signifies uncorrelated rankings. By assigning varying degrees of importance to different ranking positions, this method offers a nuanced perspective. The r_W coefficient is formally defined as Eq. (12):

$$r_W = 1 - \frac{6 \cdot \sum (x_i - y_i)^2 ((n - x_i + 1) + (n - y_i + 1))}{n \cdot (n^3 + n^2 - n - 1)} \tag{12}$$

3. Study case

This research compares objective weighting methods with the Stochastic Identification of Weights (SITW) technique in the context of identifying multi-criteria decision analysis (MCDA) models. The classic MCDA method, known as TOPSIS, was employed to generate rankings for predefined alternatives and criterion weights. The MCDA methods implemented in this investigation were sourced from the pymcdm library [19]. Spearman's weighted correlation coefficient served as a metric to assess the fitting accuracy of the MCDA models. The benchmark function utilized in this study was the Thrombolysis In Myocardial Infarction Risk Index (TRI), designed to evaluate patients with acute coronary insufficiency. The TRI function served as a benchmark to evaluate the effectiveness of various weight selection approaches. The expression for the TRI function is as follows:

$$TRI = \frac{HR \cdot A^2}{100 \cdot SBP} \tag{13}$$

where A is the patient's age, HR is the number of heartbeats per minute, and SBP is the systolic blood pressure measured in millimeters of mercury column.

For this study, randomly generated sets of alternatives were used for three criteria that corresponded to the following parameters of the TRI function, i.e., C_1 - HR (heart rate), C_2 - A (age), and C_3 - SBP (systolic blood pressure). To determine these sets, criterion intervals are defined as follows: C_1 belongs to the interval [60, 100], C_2 belongs to the interval [40, 60], and C_3 belongs to the interval [90, 180]. Based on the methodology presented in the paper [20], an example set of 10 alternatives was generated, shown in the Table 1.

Table 1
Example decision matrix for the problem of evaluating patients suffering from acute coronary insufficiency

C_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
C_1	98	80	80	72	84	77	71	76	79	94
C_2	58	54	58	43	44	56	59	50	54	52
C_3	151	131	95	146	91	164	162	171	159	105

For study purposes, the focus will be on examining three weight selection methods: the Entropy method, the STD method, and the Stochastic Identification of Weights (SITW) method. Since the SITW method needs previously evaluated samples for its weighting mechanism, it was decided to use 160 randomly generated alternatives, and their evaluations were derived from the TRI function. Since the SITW method is based on rankings, the ratings derived from the TRI function were converted to rankings. For the SITW method, TOPSIS was used as the base method, while Particle Swarm Optimization (PSO) was used as the optimization method. For the re-identification of the weights, the PSO parameters were set as default settings from the MEALPY library [21]. In contrast, two parameters were modified, i.e., the number of epochs - 1000 and the number of particles - 100. For the SITW approach, the following weights were identified: $w_1 = 0.28999945$, $w_2 = 0.36769221$ and $w_3 = 0.34230833$. On the other hand, the optimization process concerning the course of the fitness function is presented in Figure 1.

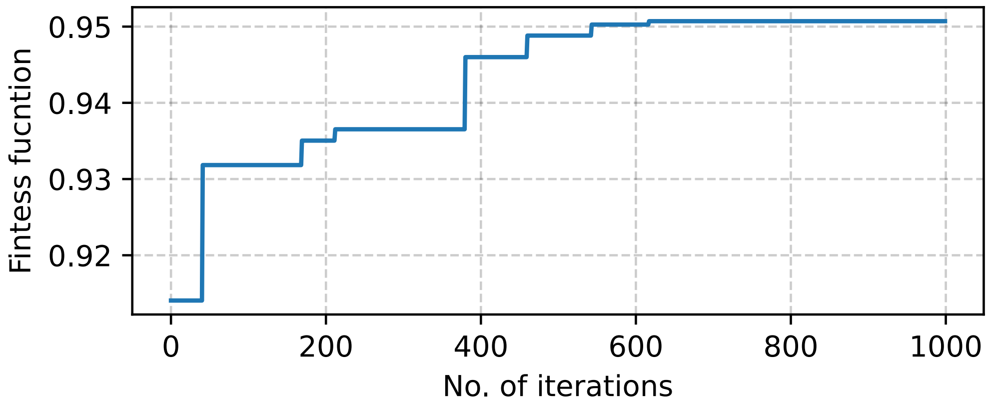

Fig. 1. The learning flow of the SITW method.

Once the criterion weights were obtained using the SITW approach, they were contrasted with the equal weights approach. In this case, the TOPSIS method was used to transfer weights and decision matrices to obtain rankings. This study was conducted on 1000 random sets containing $\{10, 25, 50\}$ alternatives. Spearman's weighted correlation coefficient (r_w) was used to compare the resulting rankings from the TOPSIS method and the TRI reference approach.

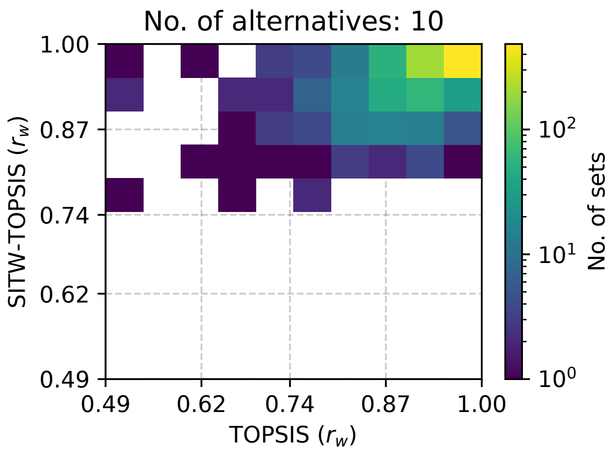

Fig. 2. Two-dimensional histogram showing the distribution of r_w coefficient for rankings derived from TOPSIS and Equal and SITW weighting methods for 10 alternatives

A comparison of reference rankings with the SITW-TOPSIS and TOPSIS approaches for 10 alternatives randomly generated 1,000 times is shown in Figure 2. It can be observed that the SITW-TOPSIS approach performs much better than the classical TOPSIS approach with equal weights. The range of obtained coefficient values r_w for the SITW-TOPSIS approach is [0.74, 1.0]. In contrast, the range of r_w coefficient values for the TOPSIS approach with equal weights is [0.49, 1.0]. In contrast, the most robust clustering for both studied approaches of the r_w coefficient comes from the range [0.87, 1.00].

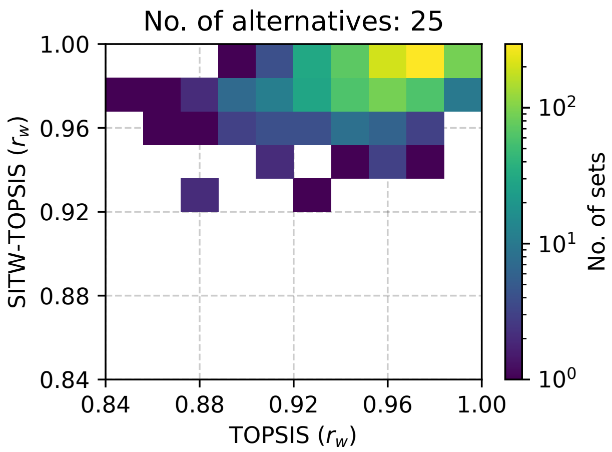

Fig. 3. Two-dimensional histogram showing the distribution of r_w coefficient for rankings derived from TOPSIS and Equal and SITW weighting methods for 25 alternatives

Figure 3 compares reference rankings with the SITW-TOPSIS and TOPSIS approaches for 25 alternatives randomly generated 1000 times. A significant improvement in the ranking quality of the TOPSIS method using equal weights was observed in this analysis. Nevertheless, the SITW-TOPSIS approach achieves significantly higher r_w coefficient values in cases where TOPSIS using equal weights achieves them significantly lower. Nevertheless, in most cases, both approaches achieve equally high r_w coefficient values in the range of 0.96 to 0.98. Notably, most of the r_w values obtained are significantly smaller than those of the 10 alternatives. Nevertheless, the standard deviation for both approaches is also more minor.

Figure 4 compares reference rankings with the SITW-TOPSIS and TOPSIS approaches for 50 alternatives randomly generated 1,000 times. Both approaches perform very well in two-dimensional histogram, identifying TOPSIS models close to reflecting the TRI function. Nevertheless, there is a noticeable trend that the rankings obtained from both approaches decrease as the number of alternatives increases. It is also worth noting that the highest r_w values for the SITW-TOPSIS and TOPSIS approaches with equal weights were obtained from 0.96 to 0.98. However, again, the overall range of accepted r_w values narrowed for the approaches, where it is 0.97 to 1.0 for SITW-TOPSIS and 0.92 to 1.0 for TOPSIS with equal weights.

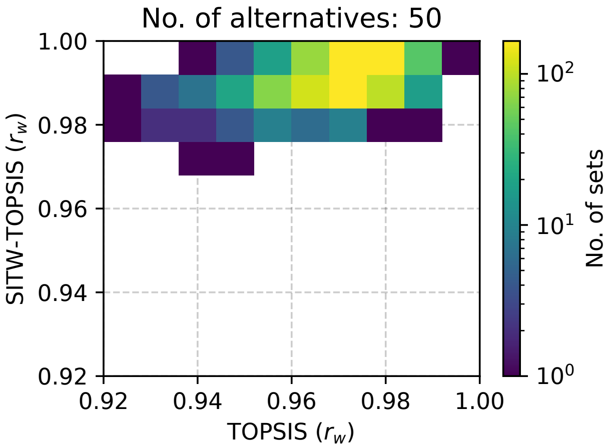

Fig. 4. Two-dimensional histogram showing the distribution of r_w coefficient for rankings derived from TOPSIS and Equal and SITW weighting methods for 50 alternatives

In addition to comparing the classic TOPSIS approach with equal weights and the SITW-TOPIS approach, objective weighting methods were analyzed. As in the previous study, 1000 decision matrices containing 5, 10, 15, 25, 50 alternatives were randomly drawn. The box plots in Figure 5 compare the comparison. Analyzing the overall comparison, the SITW-TOPIS approach coped best with the problem of identifying the reference model, i.e., TRI. It showed the highest averages for the tested decision matrix sizes and fewer outliers than the rest of the tested approaches. Equal weights did equally well in the case of identifying TOPSIS models that are close in similarity to the reference TRI model. However, for the identification of TOPSIS models using weights based on measures of information, i.e., Entropy and Standard Deviation, mapping the TRI method could have been more effective. Mostly, the values of both of these approaches clustered for the tested sizes of the number of alternatives around the value of 0.75 r_w, which does not indicate high identification accuracy.

Table 2 presents detailed comparative statistics of TOPSIS approaches using different weights with the reference TRI function. For the Entropy-TOPSIS method, it can be seen that for sets containing 25 and 50 alternatives, the values of the rw coefficient oscillate around 0.77 and 0.78, suggesting the moderate effectiveness of this method in identifying models. However, the standard deviation of these values is relatively high, which may indicate some variability in the effectiveness of this method depending on the specific dataset. For the SITW-TOPSIS method, on the other hand, we observe significantly higher values of the rw coefficient, especially for larger datasets. For a dataset containing 50 alternatives, the average rw value exceeds 0.99, indicating that this method is highly effective in identifying models. In addition, the standard deviation for this method is relatively low, suggesting the stability of its results. Similarly, for the STD-TOPSIS method, the rw values also show an increasing trend with the increasing number of alternatives, reaching an average value of about 0.70 for a set containing 50 alternatives. However, compared to the SITW-TOPSIS method, the average rw values are lower, which may indicate that this method is less effective in identifying models.

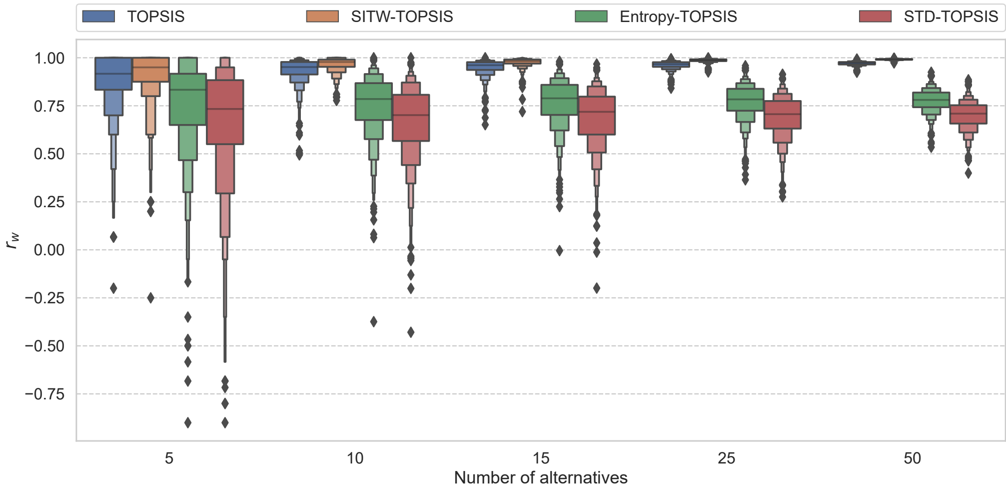

Fig. 5. Box plots of rank comparisons from the TOPSIS approach with the TRI function using the r_w coefficient for different sizes of 1000 random sets of alternatives

Table 2
Statistics of comparison of rankings from TOPSIS approaches with TRI function using r_w coefficient for different sizes of 1000 random sets of alternatives

Method	No. of. alts.	Mean	STD	Min	Max
Entropy-TOPSIS	5	0.75163	0.26156	-0.90000	1.00000
	10	0.75526	0.15948	-0.37411	1.00000
	15	0.76637	0.12638	-0.00424	0.98170
	25	0.77166	0.09119	0.36429	0.95831
	50	0.77589	0.06017	0.53423	0.92468
SITW-TOPSIS	5	0.90695	0.13900	-0.25000	1.00000
	10	0.96456	0.03794	0.77741	1.00000
	15	0.97658	0.02266	0.71942	1.00000
	25	0.98586	0.00888	0.92583	0.99864
	50	0.99112	0.00336	0.97325	0.99773
STD-TOPSIS	5	0.65622	0.32171	-0.90000	1.00000
	10	0.66751	0.19499	-0.42920	1.00000
	15	0.68753	0.15510	-0.19933	0.96719
	25	0.69324	0.11098	0.27544	0.91420
	50	0.70046	0.07538	0.39946	0.88524
TOPSIS	5	0.87658	0.16781	-0.20000	1.00000
	10	0.93403	0.06500	0.49421	1.00000
	15	0.95054	0.03969	0.65134	1.00000
	25	0.96220	0.02055	0.84139	0.99388
	50	0.97057	0.01021	0.92437	0.99331

4. Conclusions

A study on the identification of weights in multi-criteria decision analysis (MCDA) models was conducted in this benchmark analysis. The TRI function for evaluating patients with acute coronary insufficiency was used as the reference model. The evaluated approaches were: TOPSIS (use of equal weights), SITW-TOPSIS, Entropy-TOPSIS, and STD-TOPSIS. The study results indicated that the best approach for identifying multi-criteria weights is the SITW-TOPSIS method. It uses information from previously evaluated alternatives, which allows better matching of weights compared to approaches based on measures of information. The analysis confirmed the high performance of the SITW-TOPSIS method and the moderate performance of the Entropy-TOPSIS and STD-TOPSIS methods.

Future research would need to focus around a broader analysis of selected approaches with different reference models. In this case, the Fuzzy Reference Model or other benchmark functions could be used. In addition, this study would need to be expanded to include cases with uncertainty.

Acknowledgement

The work was supported by the National Science Centre, Poland, Decision number UMO-2021/41/B/HS4/01296.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Özceylan, E., Çetinkaya, C., Erbaş, M., & Kabak, M. (2016). Logistic performance evaluation of provinces in Turkey: A GIS-based multi-criteria decision analysis. *Transportation Research Part A: Policy and Practice*, *94*, 323–337. https://doi.org/10.1016/j.tra.2016.09.020

[2] Longaray, A., Ensslin, L., Ensslin, S., Alves, G., Dutra, A., & Munhoz, P. (2018). Using MCDA to evaluate the performance of the logistics process in public hospitals: the case of a Brazilian teaching hospital. *International Transactions in Operational Research*, *25*(1), 133–156. https://doi.org/10.1111/itor.12387

[3] Uhde, B., Andreas Hahn, W., Griess, V. C., & Knoke, T. (2015). Hybrid MCDA methods to integrate multiple ecosystem services in forest management planning: a critical review. *Environmental management*, *56*, 373–388. https://doi.org/10.1007/s00267-015-0503-3

[4] Wątróbski, J., & Jankowski, J. (2016). Guideline for MCDA method selection in production management area. *New Frontiers in Information and Production Systems Modelling and Analysis: Incentive Mechanisms, Competence Management, Knowledge-based Production*, 119–138. https://doi.org/10.1007/978-3-319-23338-3_6

[5] Angelis, A., & Kanavos, P. (2017). Multiple criteria decision analysis (MCDA) for evaluating new medicines in health technology assessment and beyond: the advance value framework. *Social Science & Medicine*, *188*, 137–156. https://doi.org/10.1016/j.socscimed.2017.06.024

[6] Badia, X., Chugani, D., Abad, M. R., Arias, P., Guillén-Navarro, E., Jarque, I., Posada, M., Vitoria, I., & Poveda, J. L. (2019). Development and validation of an MCDA framework for evaluation and decision-making of orphan drugs in Spain. *Expert Opinion on Orphan Drugs*, *7*(7-8), 363–372. https://doi.org/10.1080/21678707.2019.1652163

[7] Deshpande, P. C., Skaar, C., Brattebø, H., & Fet, A. M. (2020). Multi-criteria decision analysis (MCDA) method for assessing the sustainability of end-of-life alternatives for waste plastics: A case study of Norway. *Science of the total environment*, *719*, 137353. https://doi.org/10.1016/j.scitotenv.2020.137353

[8] Puška, A., Stević, Ž., & Pamučar, D. (2022). Evaluation and selection of healthcare waste incinerators using extended sustainability criteria and multi-criteria analysis methods. *Environment, Development and Sustainability*, 1–31. https://doi.org/10.1007/s10668-021-01902-2

[9] Colapinto, C., Jayaraman, R., Ben Abdelaziz, F., & La Torre, D. (2020). Environmental sustainability and multifaceted development: multi-criteria decision models with applications. *Annals of Operations Research*, *293*(2), 405–432. https://doi.org/10.1007/s10479-019-03403-y

[10] Lahdelma, R., Miettinen, K., & Salminen, P. (2005). Reference point approach for multiple decision makers. *European Journal of Operational Research*, *164*(3), 785–791. https://doi.org/10.1016/j.ejor.2004.01.030

[11] Hatefi, S., & Torabi, S. A. (2010). A common weight MCDA–DEA approach to construct composite indicators. *Ecological Economics*, *70*(1), 114–120. https://doi.org/10.1016/j.ecolecon.2010.08.014

[12] Zborowski, M., & Chmielarz, W. (2023). Sensitivity analysis of the criteria weights used in selected MCDA methods in the multi-criteria assessment of banking services in Poland in 2022. *2023 18th Conference on Computer Science and Intelligence Systems (FedCSIS)*, 1217–1222. https://doi.org/10.15439/2023F3745

[13] Martin, T. G., Burgman, M. A., Fidler, F., Kuhnert, P. M., Low-Choy, S., McBride, M., & Mengersen, K. (2012). Eliciting expert knowledge in conservation science. *Conservation Biology*, *26*(1), 29–38. https://doi.org/10.1111/j.1523-1739.2011.01806.x

[14] Paradowski, B., Shekhovtsov, A., Bączkiewicz, A., Kizielewicz, B., & Sałabun, W. (2021). Similarity Analysis of Methods for Objective Determination of Weights in Multi-Criteria Decision Support Systems. *Symmetry*, *13*(10), 1874. https://doi.org/10.3390/sym13101874

[15] Kizielewicz, B., Paradowski, B., Więckowski, J., & Sałabun, W. (2022). Identification of weights in multi-cteria decision problems based on stochastic optimization. *AMCIS 2022 Proceedings*, (17). https://aisel.aisnet.org/amcis2022/sig_odis/sig_odis/17

[16] Yoon, K. P., & Kim, W. K. (2017). The behavioral TOPSIS. *Expert Systems with Applications*, *89*, 266–272. https://doi.org/10.1016/j.eswa.2017.07.045

[17] Kizielewicz, B., Więckowski, J., & Wątrobski, J. (2021). A study of different distance metrics in the TOPSIS method. *Intelligent Decision Technologies: Proceedings of the 13th KES-IDT 2021 Conference*, 275–284. https://doi.org/10.1007/978-981-16-2765-1_23

[18] Dancelli, L., Manisera, M., & Vezzoli, M. (2013). On two classes of Weighted Rank Correlation measures deriving from the Spearman's ρ. In *Statistical models for data analysis* (pp. 107–114). Springer. https://doi.org/10.1007/978-3-319-00032-9_13

[19] Kizielewicz, B., Shekhovtsov, A., & Sałabun, W. (2023). pymcdm—The universal library for solving multi-criteria decision-making problems. *SoftwareX*, *22*, 101368. https://doi.org/10.1016/j.softx.2023.101368

[20] Sałabun, W., & Piegat, A. (2017). Comparative analysis of MCDM methods for the assessment of mortality in patients with acute coronary syndrome. *Artificial Intelligence Review*, *48*, 557–571. https://doi.org/10.1007/s10462-016-9511-9

[21] Van Thieu, N., & Mirjalili, S. (2023). MEALPY: An open-source library for latest meta-heuristic algorithms in Python. *Journal of Systems Architecture*, *139*, 102871. https://doi.org/10.1016/j.sysarc.2023.102871